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International Workshop on Mathematical Methods for Cryptography

Recent Advances in Doppler Resilient Sequence Design and Applications

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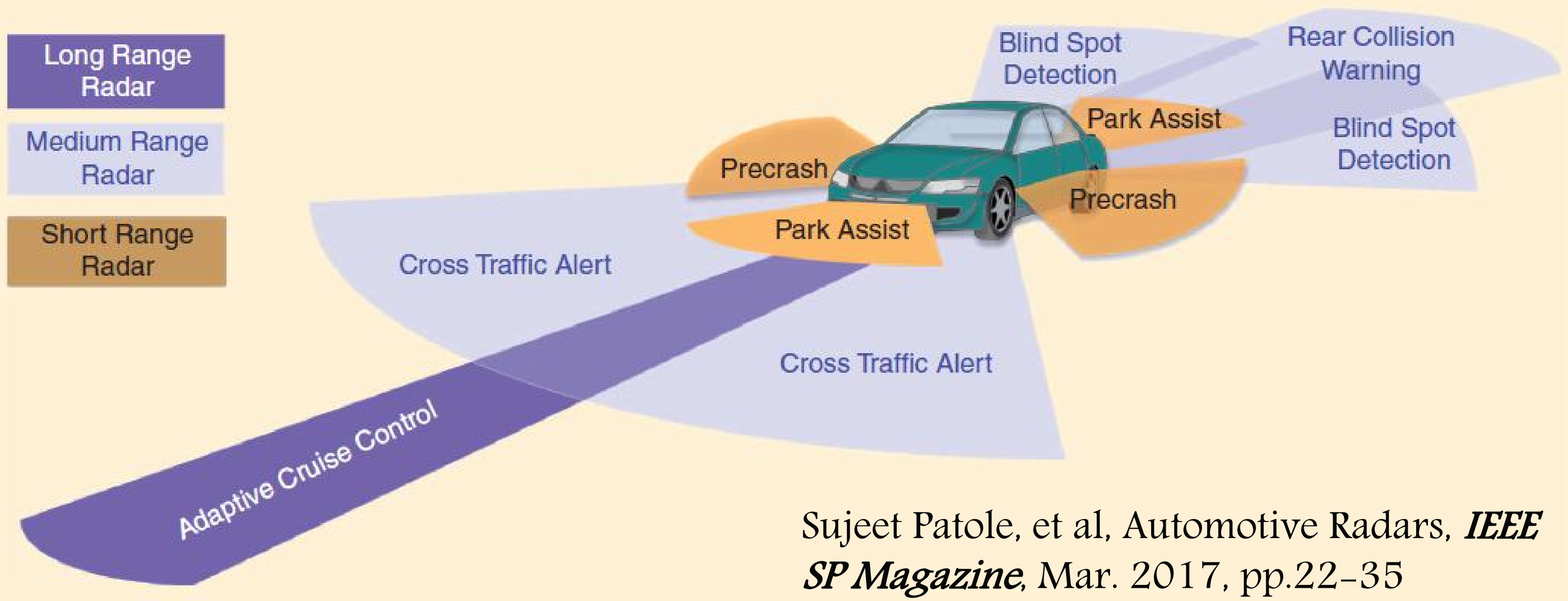
Outline

- Automotive Radars and Related Signals
- Pulse Compression and Phase Coding
- Doppler Resilient Sequences (DRS)
- DRS Design based on Z-Ambiguity
- Seqs for Optimized AF & PAPR in CR

Self-Driving Cars & Radars

- Advances in circuit tech reinforced by new signal processing algorithms, machine learning, artificial intelligence, and computervision tech have made self-driving cars a reality.
- Self-driving cars and advanced driver assistant systems (ADASs) consists of mainly **automotive radars**, lidar (light detection and ranging), ultrasound, cameras, and **V2X comms**.



Sujeet Patole, et al, Automotive Radars, ***IEEE SP Magazine***, Mar. 2017, pp.22-35

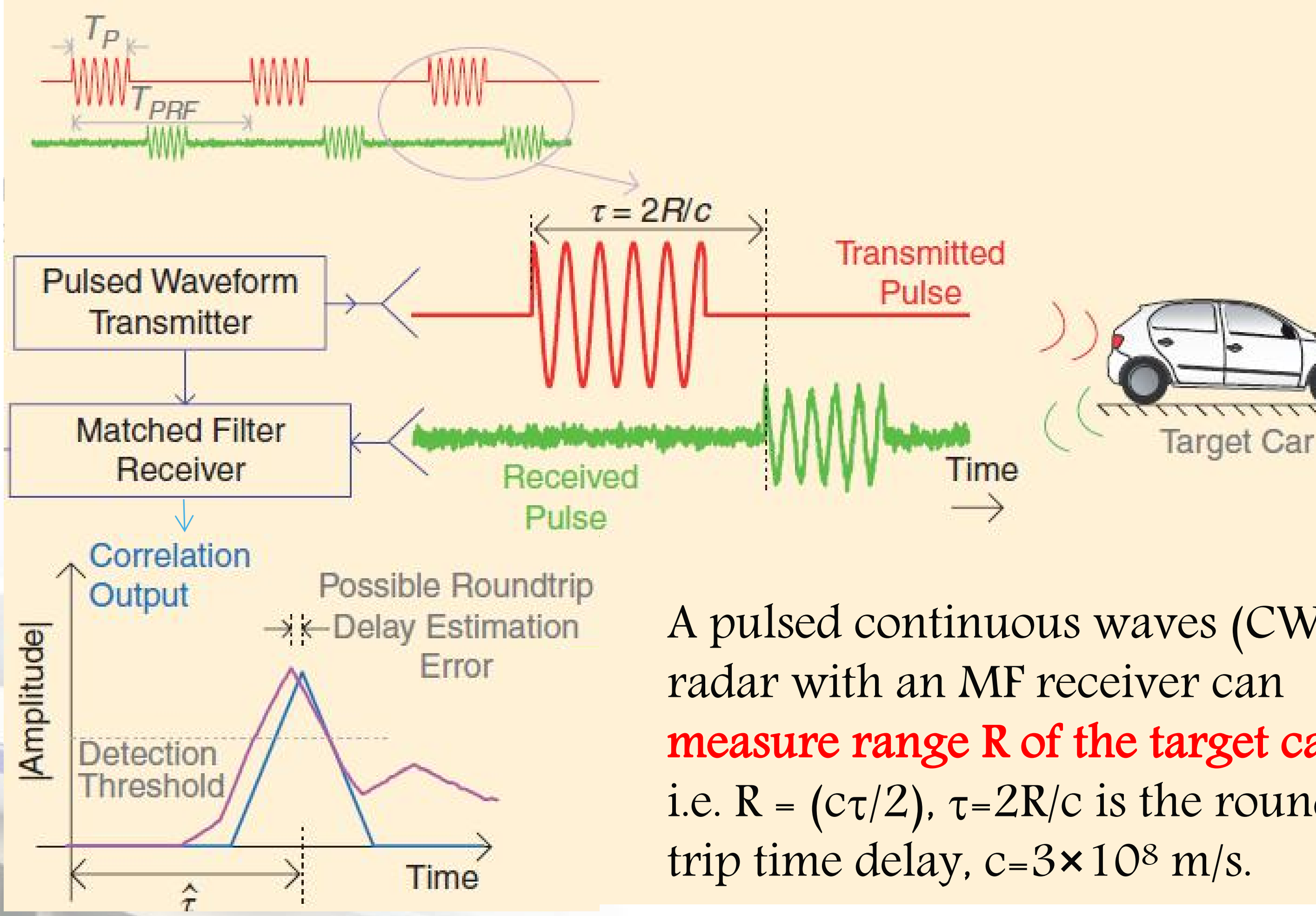
Classification of Automobile Radars

Automotive radars based on range measurement capability

Radar Type	Long-Range Radars	Medium-Range Radars	Short-Range Radars
Range (m)	10–250	1–100	0.15–30
Azimuthal field of view (deg.)	±15°	±40°	±80°
Elevation field of view (deg.)	±5°	±5°	±10°
Applications	Automotive cruise control	Lane-change assist, cross-traffic alert, blind-spot detection, rear-collision warning	Park assist, obstacle detection, precrash

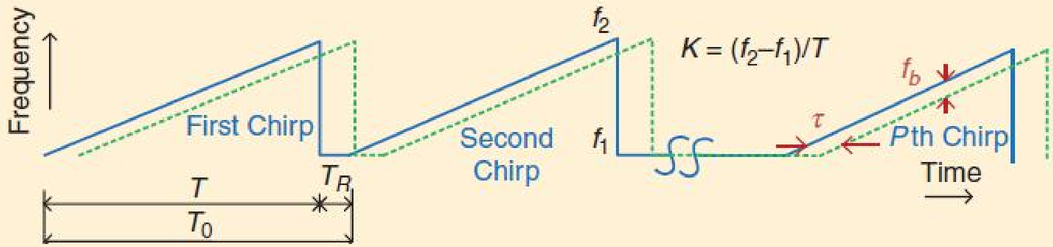
Note: An automobile radar is designed to extract location, range, velocity and radar cross section (RCS)] about targets, typically operating at mm-wave bands 24–29GHz and 76–81GHz bands (other radars may use 3 MHz to 300 GHz)

A Pulsed CW Radar with MF Receiver

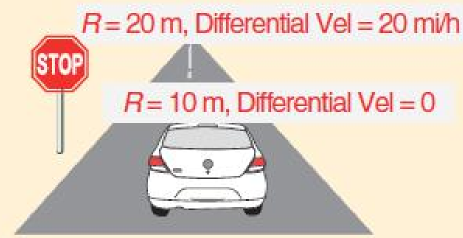


A pulsed continuous waves (CW) radar with an MF receiver can **measure range R of the target car**, i.e. $R = (c\tau/2)$, $\tau=2R/c$ is the round-trip time delay, $c=3\times10^8$ m/s.

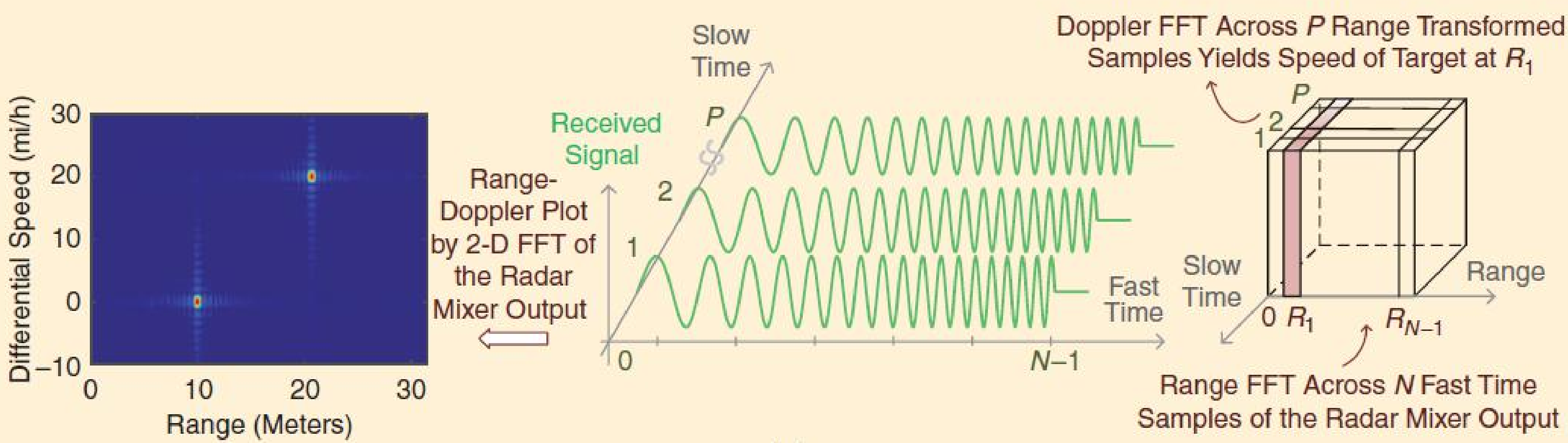
Frequency Modulated (FM) CW Radar



A spectrogram of an FMCW waveform with modulation constant K



Typical traffic scenario



A 2-D joint range-Doppler estimation with 77-GHz FMCW radar

The reflected waves are delayed by time $\tau = 2(R \pm vt)/c$. The time dependent delay term causes a frequency shift in the received wave known as the Doppler shift $f_d = \pm 2v/\lambda = \pm 2vf_C/c$.

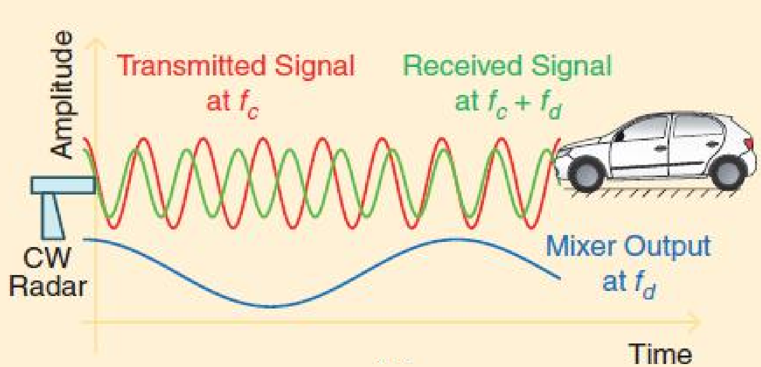
Radar Waveforms
CW, pulsed and frequency

Waveform Type	Transmit Waveform s(t)	Detection Principle	Resolution
CW	$e^{j2\pi f_c t}$	Conjugate mixing	$\triangle f_d = 1/T$
Pulsed CW	$\Pi(T_p)\, e^{j2\pi f_c t}$	Correlation	$\triangle R = cT_p/2$ $\triangle f_d = 1/T_p$
FMCW	$e^{j2\pi(f_c + 0.5Kt)t}$, $K = \frac{B}{T_0}$	Conjugate mixing	$\triangle R = c/2B$ $\triangle f_d = 1/PT_0$
SFCW	$e^{j2\pi f_n t}$, $f_n = f_c + (n-1)\triangle f$	Inverse Fourier transform	$\triangle R = c/2B$ $\triangle f_d = 1/PT_0$
OFDM	$\sum_{n=0}^{N-1} I(n)\, e^{2\pi(f_c + n\triangle f)t}$	Frequency domain channel estimation	$\triangle R = c/N\Delta f$ $\triangle f_d = 1/PT_N$

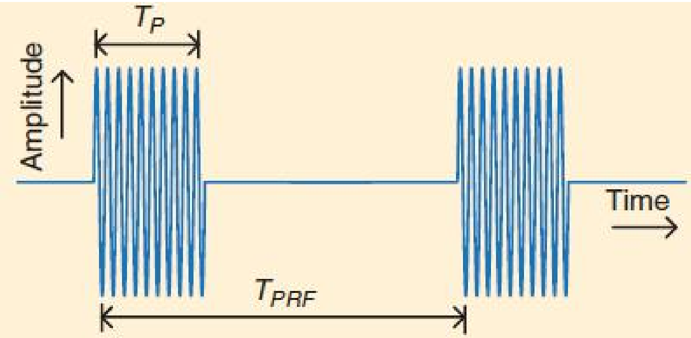
B denotes bandwidth of the radar. T is the amount of time for which data is captured.
N stands for a number of samples in CW and number of carriers in OFDM.
$\Pi(T_p)$ is rectangular pulse of duration T_p. P is number of FM/SF-CW or OFDM blocks of duration T_0 and T_N, respectively.
$I(n)$ is arbitrary sequence and $\triangle f$ is carrier/frequency separation in OFDM/SFCW.

- CW(continuous wave) provides no range information
- Pulsed CW can make range-Doppler performance tradeoff
- FMCW gives both range and Doppler information
- In Stepped Freq CW (SFCW), Δf decides maximum range
- OFDM is suitable for radar & vehicular communications

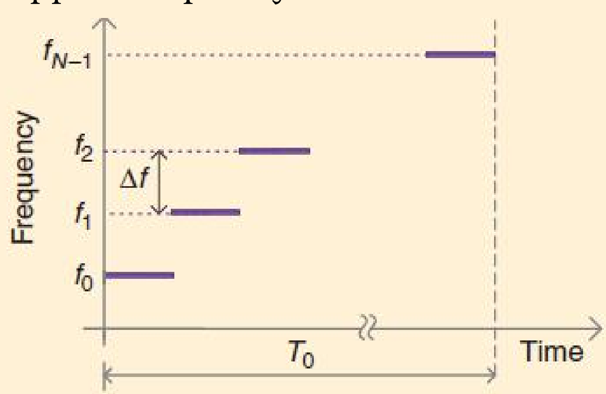
Doppler Freq Measur. by SFCW Radar



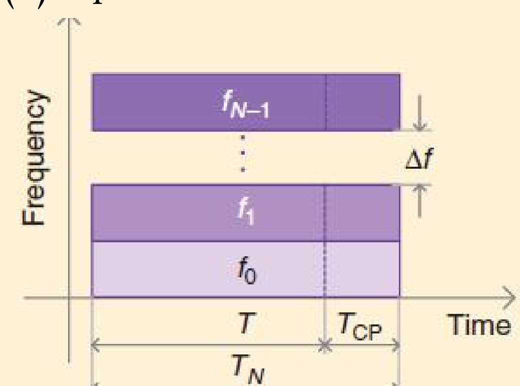
(a) Doppler frequency measurement with CW radar



(b) A pulsed CW radar waveform



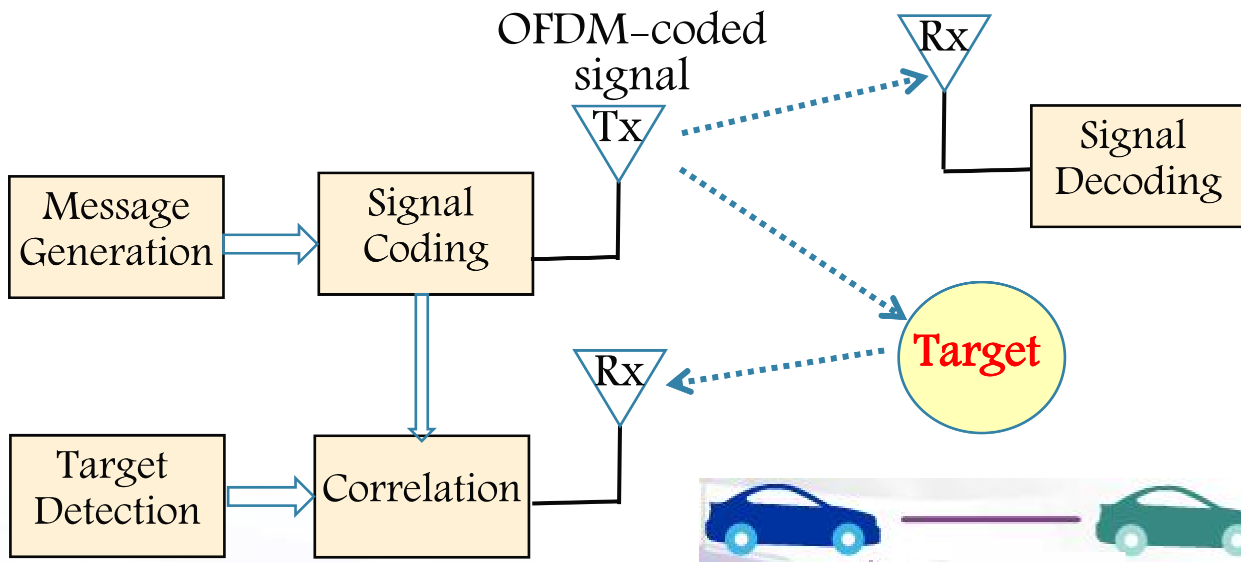
(c) An SFCW signal



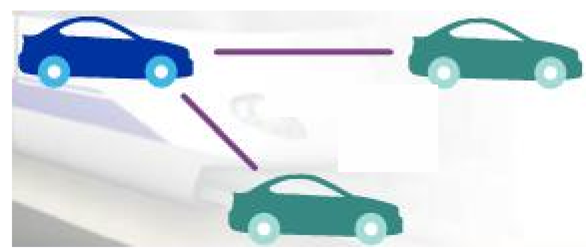
(d) An OFDM block

With the ability to measure both range and speed with high resolution, FMCW radar is widely used in the automotive industry.

OFDM-coded Radar Signals



OFDM Radar signals experience no Range-Doppler coupling, compared with LFM pulse compression.



The Radar part is formed by two co-located and co-rotating antennas for transmission and reception. The communication part uses the same tx antenna, but a remotely positioned Rx antenna.

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Radar Range Resolution

The return radar signal, r(t), is an attenuated and time-shifted copy of the original transmitted signal, s(t), plus Gaussian noise N(t). To detect the incoming signal, matched filtering is commonly used, which is optimal when a known signal is to be detected among additive white Gaussian noise, i.e. the **cross-correlation** of s(t) and r(t),

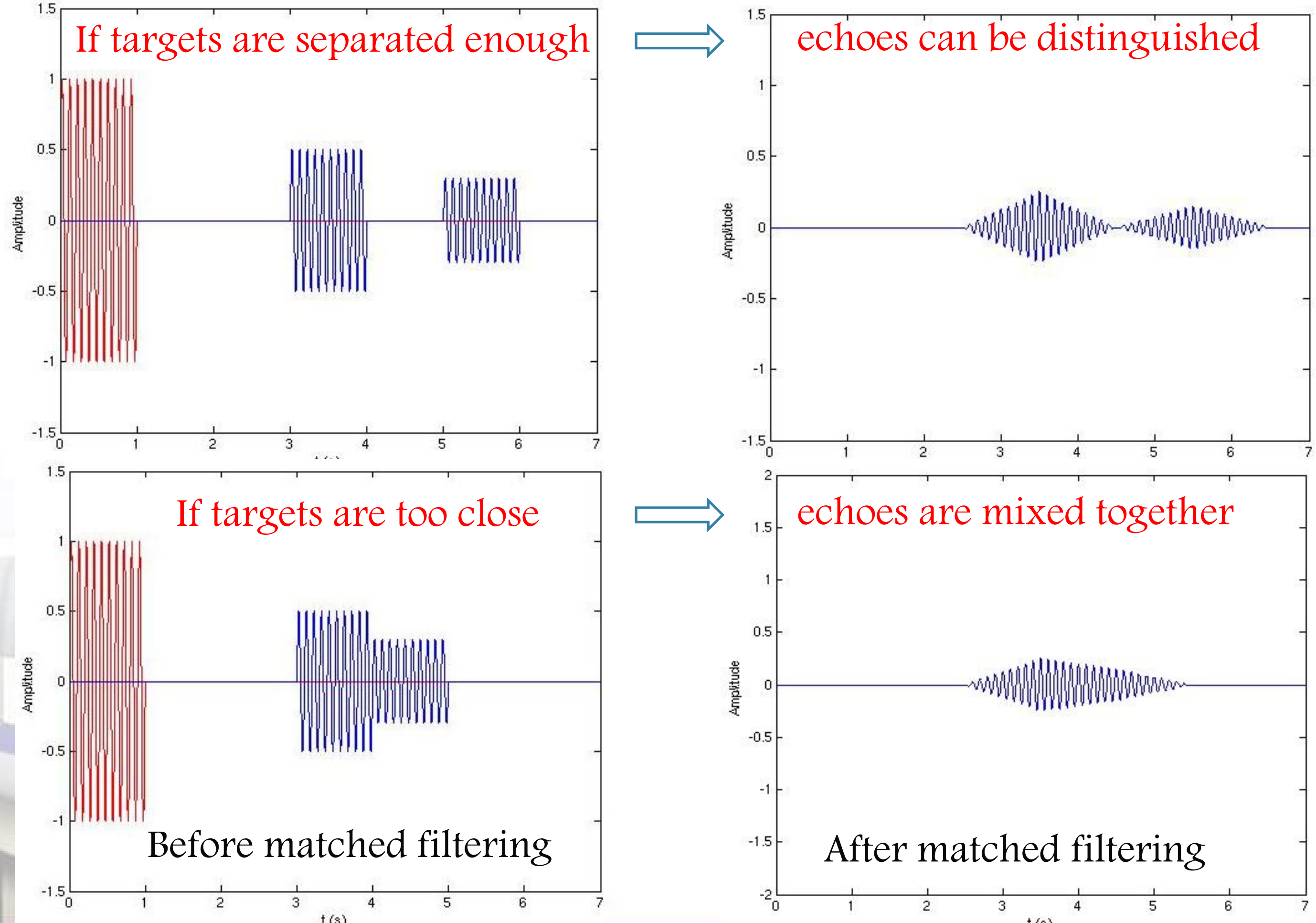
$$s(t)=\begin{cases}Ae^{2i\pi f_0 t}, & 0\le t\le T\\ 0, & otherwise\end{cases},\quad r(t)=\begin{cases}KAe^{2i\pi f_0(t-t_r)}+N(t), & t_r\le t\le t_r+T\\ N(t), & otherwise\end{cases}$$

$$\langle s,r\rangle(t)=\int_{\tau=0}^{+\infty}s^*(\tau)r(\tau+t)d\tau=KA^2\Lambda\left(\frac{t-t_r}{T}\right)e^{2i\pi f_0(t-t_r)}+N'(t)$$

Function Λ is the triangle function in [-1/2,1/2], with maximum 1 at Λ(0). Thus, the times of arrival of the two pulses must be separated by at least T so that the maxima of both pulses can be separated.
The **range resolution** with a sinusoidal pulse is **cT/2** (distance travelled by a wave during T), T is the pulse duration and, c, the speed of the wave.

Radar Range Resolution

Conclusion: to increase the resolution, **pulse length T** must be reduced.



SNR, Resolution & Pulse Compression

Required energy E to transmit signal s(t), and the SNR at receiver,

$$E = \int_0^T |s(t)|^2 dt = A^2 T, \quad E_r = \int_0^T |r(t)|^2 dt = K^2 A^2 T$$

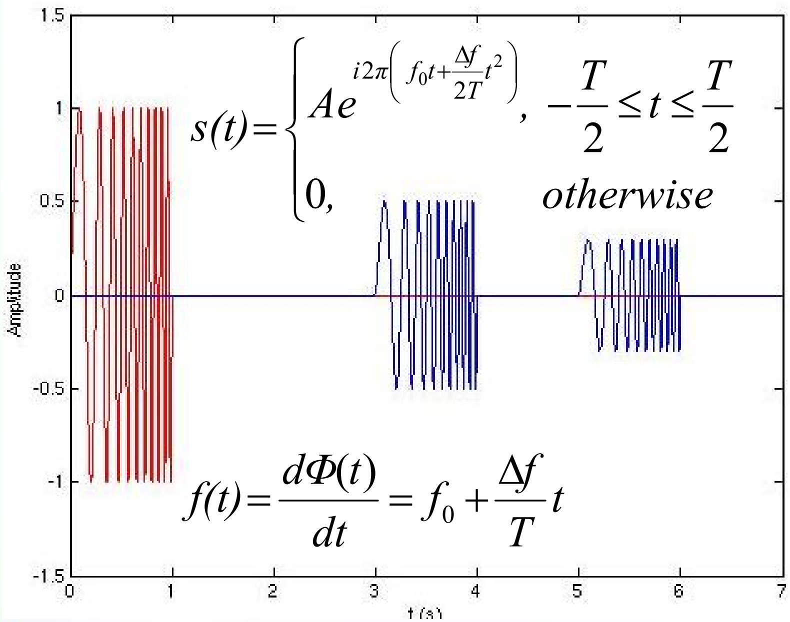
$$SNR = \frac{E_r}{\sigma^2} = \frac{K^2 A^2 T}{\sigma^2}$$

From the above SNR and the range resolution cT/2, increasing T improves the SNR, but reduces the resolution, and vice versa.

How can one have a large enough pulse (to still have a good SNR at the receiver) without poor resolution? **Pulse compression:**

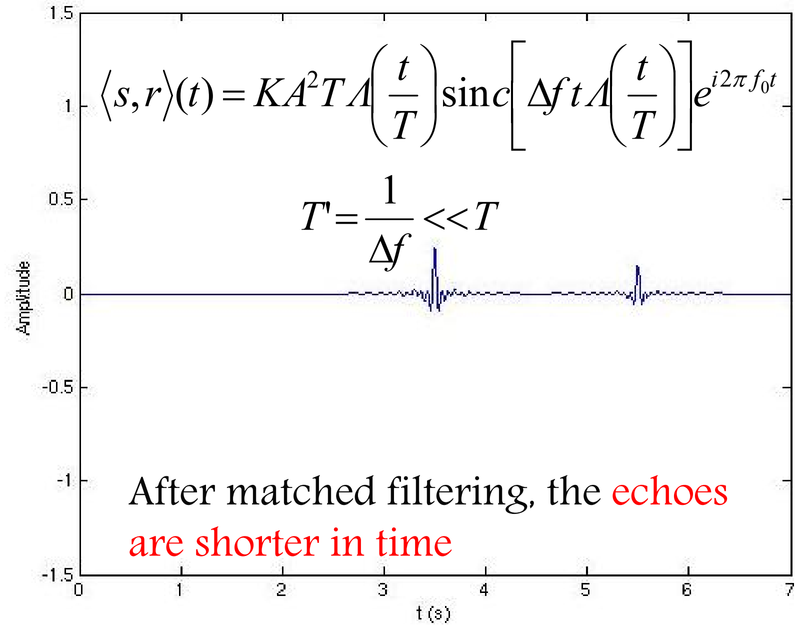
- a signal is transmitted, with a long enough length so that the energy budget is correct;
- this signal is designed so that after matched filtering, the width of the intercorrelated signals is smaller than the width obtained by the standard sinusoidal pulse, e.g. **Linear frequency modulated (LFM) pulse (or "chirp").**

LFM Resolution & Compression Ratio



$$s(t)=\begin{cases} Ae^{i2\pi\left(f_0 t+\frac{\Delta f}{2T}t^2\right)}, & -\frac{T}{2}\le t\le\frac{T}{2} \\ 0, & otherwise \end{cases}$$

$$f(t)=\frac{d\Phi(t)}{dt}=f_0+\frac{\Delta f}{T}t$$



$$\langle s,r\rangle(t)=KA^2T\Lambda\left(\frac{t}{T}\right)\mathrm{sinc}\left[\Delta f\, t\,\Lambda\left(\frac{t}{T}\right)\right]e^{i2\pi f_0 t}$$

$$T'=\frac{1}{\Delta f}<<T$$

After matched filtering, the echoes are shorter in time

- The distance resolution reachable with a LFM pulse on a bandwidth Δf is: c/(2Δf), with pulse compression ratio T/T'=T.Δf (20-30 usually)
- After pulse compression, the power of the received signal can be considered as being amplified by T.Δf.
- To deal with high instantaneous bandwidth Δf (up to 1GHz or higher), stretch processing is needed to reduce bandwidth.

Baseband Pulse & Ambiguity Function

In pulsed radar and sonar signal processing, an ambiguity function is a two-dimensional function of time delay and Doppler frequency $\chi(\tau, f)$ showing the distortion of a returned pulse due to the receiver matched filter & the Doppler shift of the return from a moving target. For a given complex baseband pulse s(t), the narrowband ambiguity function (**AF**) is given by

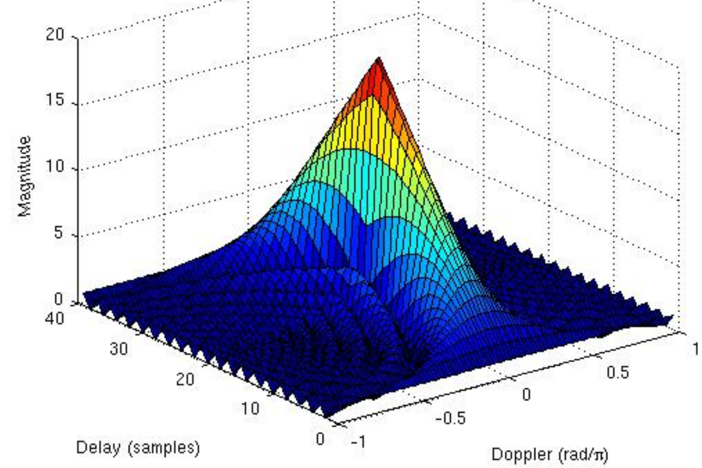
$$\chi(\tau, f) = \int_{-\infty}^{\infty} s(t) s^*(t-\tau) e^{i 2\pi f t} \, dt$$

Ideal ambiguity function

$$\chi(\tau, f) = \delta(\tau)\delta(f)$$

which is produced by ideal white noise, i.e. no ambiguities at all, not physically realizable or desirable.

AF for a square pulse



Properties of the Ambiguity Function

(1) Maximum value $|\chi(\tau, f)|^2 \le |\chi(0,0)|^2$

(2) Symmetry about the origin $\chi(\tau, f) = \exp[i2\pi\tau f]\chi^*(-\tau,-f)$

(3) **Volume invariance** $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|\chi(\tau, f)|^2 d\tau\, df \le |\chi(0,0)|^2 = E^2$

(4) Modulation by a linear FM signal

If $s(t) \to |\chi(\tau, f)|$ then $s(t)\exp[i\pi kt^2] \to |\chi(\tau, f - k\tau)|$

(5) Frequency energy spectrum $S(f)S^*(f) = \int_{-\infty}^{\infty}\chi(\tau,0)e^{-i2\pi\tau f}d\tau$

(6) Upper bounds for p>2 and lower bounds for p<2 exist for the p[th] power integrals $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|\chi(\tau, f)|^p d\tau\, df$. These bounds are sharp and are achieved if and only if s(t) is a Gaussian function.

(7) In radar, **the greater the Doppler shift, the smaller the peak of the distorted matched filter output**, and the more difficult to detect the target.

Pulse Compression by Coding

- In phase modulation, the pulse of duration T is divided into N time slots of duration T/N, each slot is coded with different phase value.

$$s(t) = \frac{1}{\sqrt{T}} \sum_{n=0}^{N-1} s_n \, rect\left[\frac{t-(n-1)T/N}{T/N}\right], \quad s_n = \exp(i\phi_n)$$

- The criteria for code design are the resolution properties of the resulting waveform (shape of the ambiguity function), frequency spectrum, and the ease with which the system can be implemented.
- The most popular phase codes are:

✓ Barker (1953) codes (up to length 13, for binary)
✓ Frank (1962), Zadoff-Chu (1963), P1, P2, Px codes (1998)
✓ HFM codes
✓ Golay complementary codes
✓ M sequences
✓ Frequency codes (Costas, Pushing sequences)

Simple Coding: Binary Barker Codes

N	+/-	Octal	PSL(dB)
2	+-	2	-6.0
2	++	3	-6.0
3	++-	6	-9.5
4	++-+	15	-12.0
4	+++-	16	-12.0
5	+++-+	35	-14.0
7	+++--+-	162	-16.9
11	+++---+--+-	3422	-20.8
13	+++++--++-+-+	17565	-22.3

Peak Sidelobe Level (PSL) ratio

$20\log_{10}(1/N)$

Merit Factor

$$MF = \frac{|\chi(0,0)|^2}{2\sum_{\tau=0}^{N-1}|\chi(\tau,0)|^2}$$

- The pulse compression ratio is lower than in the chirp case;
- The compression is very sensitive to freq changes due to the Doppler effect if that change is larger than 1/T.
- Available Barker codes are limited, i.e. only 7 lengths! Longer **"Baker"** codes can be obtained by kronecker product (nested codes).
- Lindner, Cohen, Coxson obtained minimum peak sidelobe codes up to length 69, and P Fan et al good longer codes up to length 100+.

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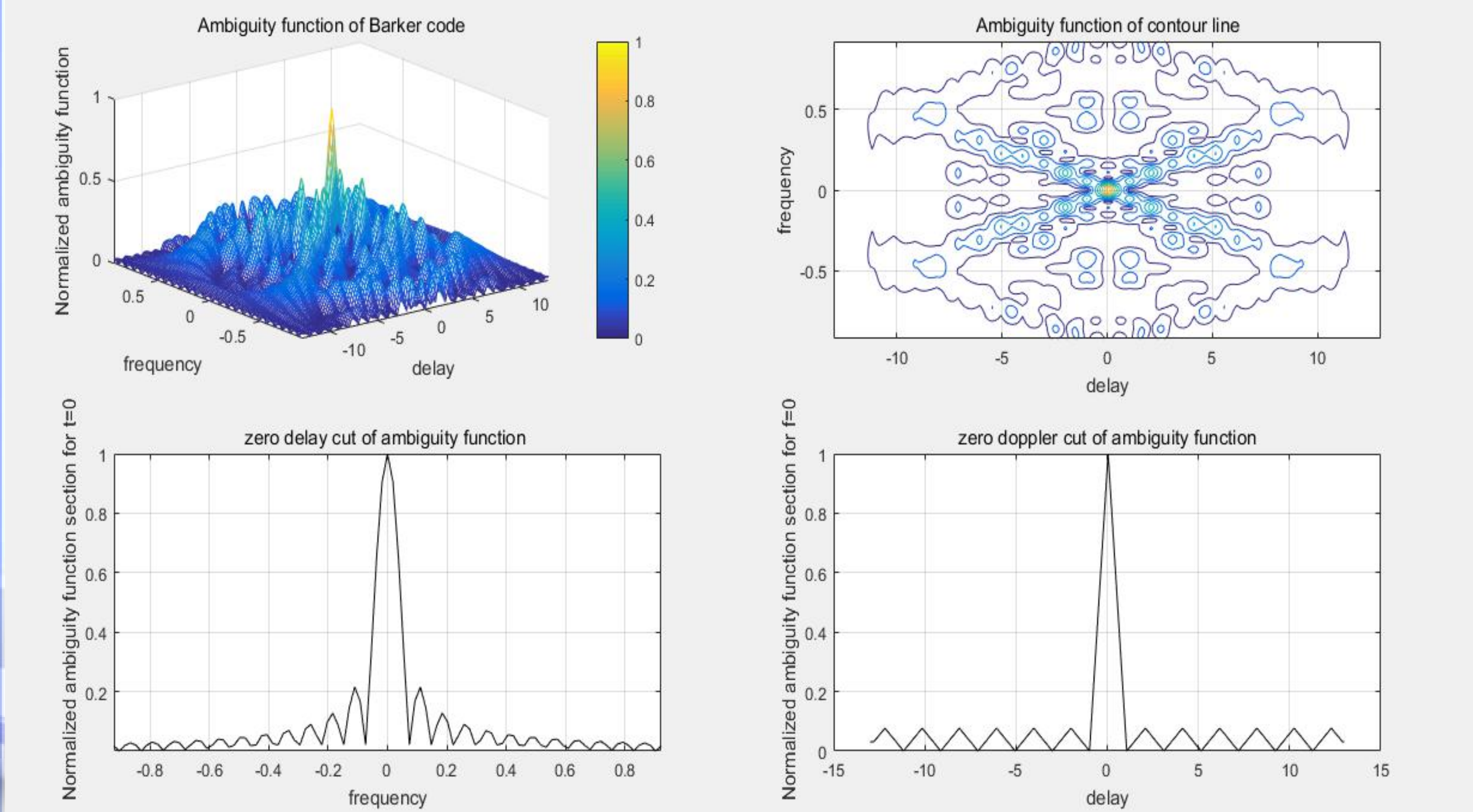
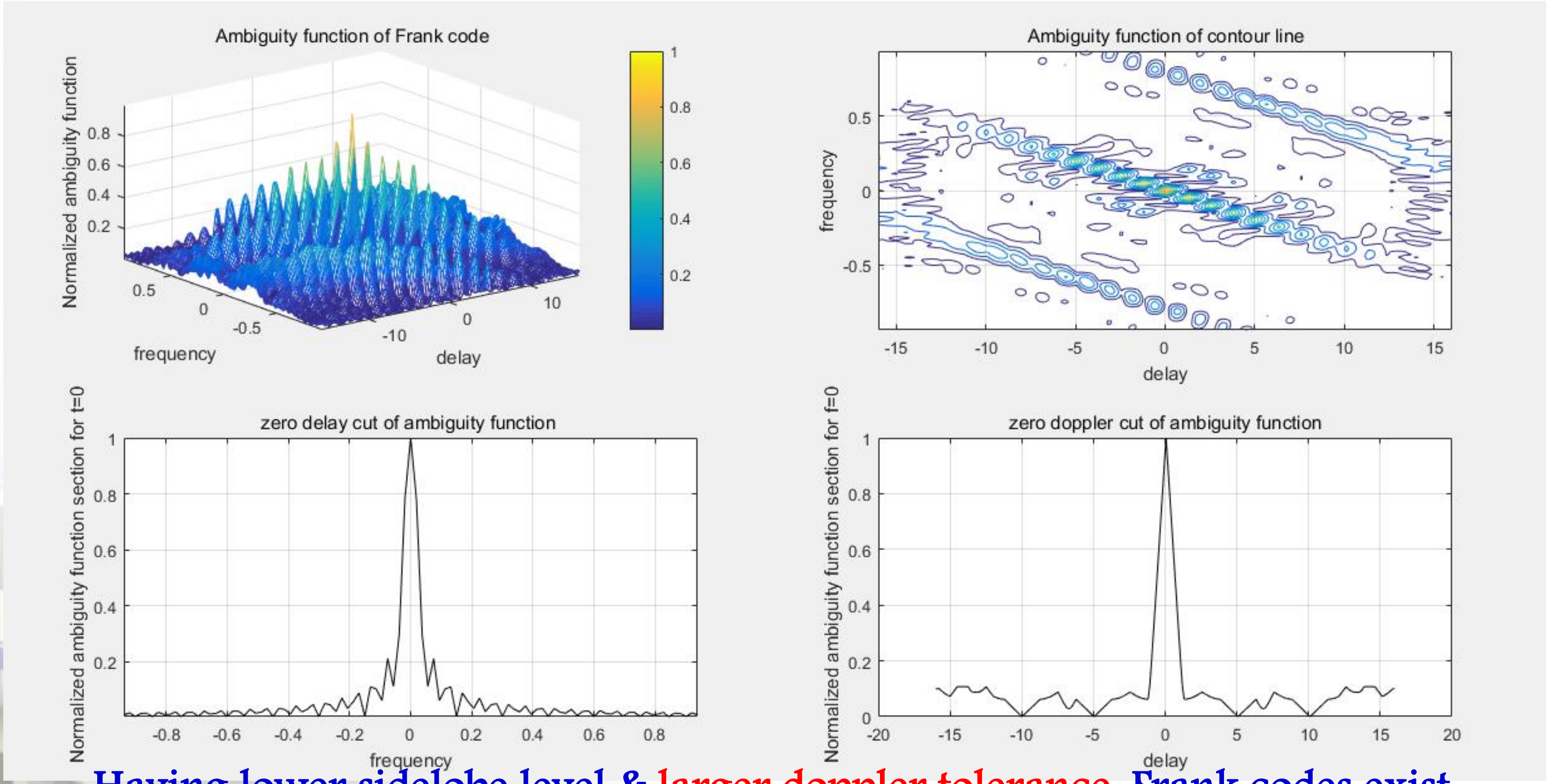
Barker Code Ambiguity Function



Drawback: once the target return is Doppler shifted, the expected sidelobes are much higher compared with the zero Doppler cut of AF.

Frank Codes

$$s_n = s_{k+(l-1)q} = \exp\left(i\frac{2\pi}{q}(k-1)(l-1) \right) = \exp(i\phi_n) \quad k,l = 1,\ldots,q;\ N = q^2$$



Having lower sidelobe level & larger doppler tolerance, Frank codes exist only for perfect square length ($N=p^2$).

Modified Frank Codes: P1, P2, Px

$$\text{Px:}\quad \phi_{k+(l-1)q} = \begin{cases} \frac{2\pi}{q}\left[(q+1)/2-k\right]\left[(q+1)/2-l\right], & q\,\text{even} \\ \frac{2\pi}{q}\left[q/2-k\right]\left[(q+1)/2-l\right], & q\,\text{odd} \end{cases} \qquad k,l=1,\ldots,q;\ N=q^2$$

- The **Px code** was shown to yield the same aperiodic peak sidelobe as the Frank code but having **lower integrated sidelobe level.**
- While the Frank code is a perfect code (having an ideal periodic autocorrelation function), the **Px code is not perfect.**
- The **P2 code** is valid only for q even and is defined exactly as the Px code for even q
- The **P1 code** phase element is defined as follows,

$$\text{P1:}\quad \phi_{k+(l-1)q} = \frac{2\pi}{q}\left[(q+1)/2-k\right]\left[(k-1)q+(l-1)\right] \quad k,l=1,\ldots,q;\ N=q^2$$

Zadoff–Chu Code

$$\text{ZC:}\quad \phi_n = \begin{cases} \dfrac{2\pi}{N} r \dfrac{(n-1)^2}{2}, & N \text{ even} \\ \dfrac{2\pi}{N} r \dfrac{(n-1)n}{2}, & N \text{ odd} \end{cases} \qquad n = 1,\ldots,N;\ (r,N) = 1$$

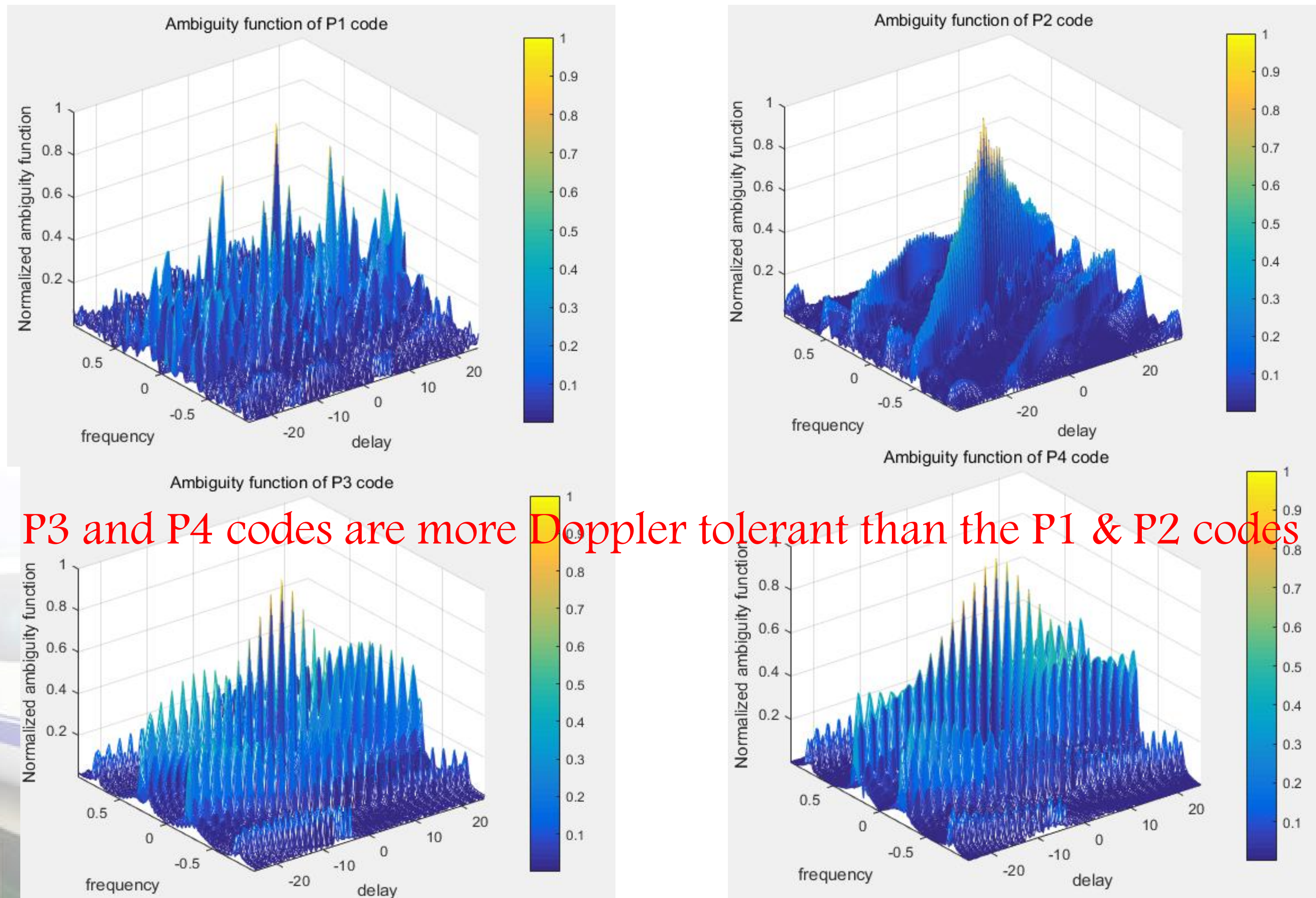
$$\text{P3:}\quad \phi_n = \frac{2\pi}{N}\frac{(n-1)^2}{2} \qquad n = 1,\ldots,N$$

$$\text{P4:}\quad \phi_n = \frac{2\pi}{N}(n-1)\frac{(n-1-N)}{2} \qquad n = 1,\ldots,N$$

$$\text{Golomb:}\quad \phi_n = \frac{2\pi}{N} r \frac{(n-1)n}{2} \qquad n = 1,\ldots,N;\ (r,N) = 1$$

The P3, P4, and Golomb polyphase codes are specific cyclically shifted and decimated versions of the Zadoff–Chu (ZC) code.

Ambiguity Function of P1/P2/P3/P4 Codes

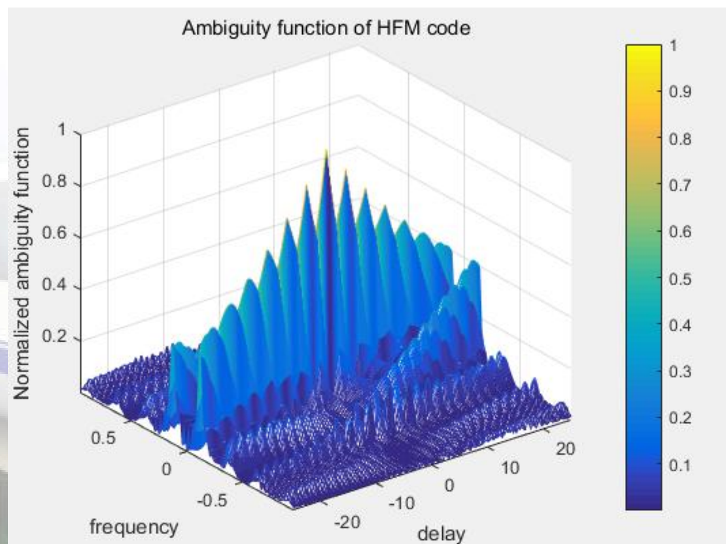


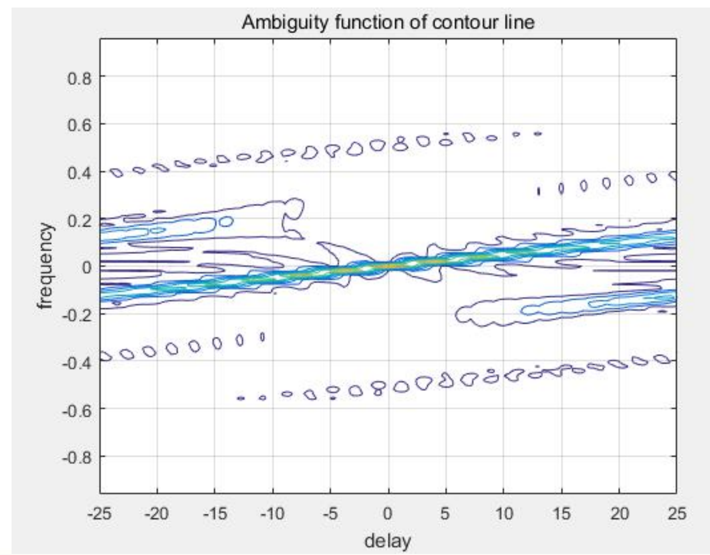
P3 and P4 codes are more Doppler tolerant than the P1 & P2 codes

HFM Codes

$$\phi_{\mathrm{n}} = \pi \log\left[1-(n-1)\beta / M\right] / \alpha, \quad n = 1,\ldots, N$$

The peak value of **hyperbolic frequency modulation (HFM)** polyphase codes, derived from the step appropriation of the face curve of the hyperbolic modulated chirp signal., degrades much slower and the range solution as well as maximum sidelobe level are almost constant when Doppler frequency increases, optimized β=0.4 & $\alpha \approx 0.2643/N$.





OFDM-Coded Radar Signals

$$s(t) = e^{i2\pi f_0 t} \sum_{i=1}^{noc} d_i \, e^{i2\pi f_i t} p(t)$$

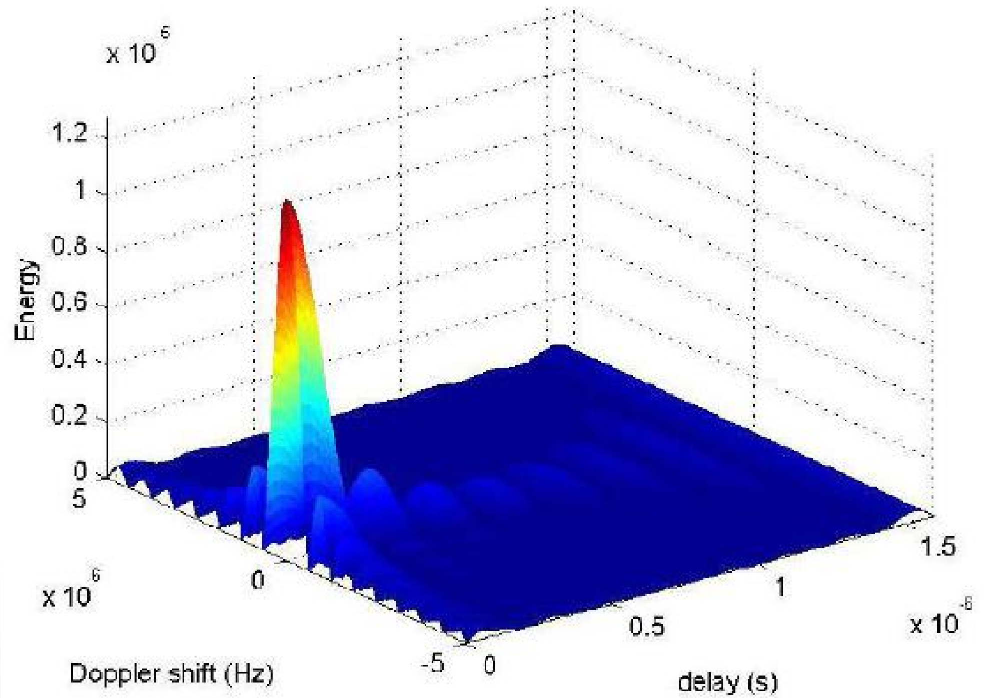
where noc is the number of freq carriers, f_0 is the center freq, and p(t) is the shaping function of duration T=noc/B, f_i=i B/noc, B is the available bandwidth; d_i is the data bit, e.g. BPSK or QPSK coded.

- The OFDM signals can result in a pulse compression ratio of up to noc. The range resolution is therefore improved without degrading the Doppler resolution.
- In order to be able to detect a larger range of target speeds, a compression filter bank should be used.
- OFDM Radar signals do not experience Range-Doppler coupling which is the main disadvantage of pulse compression using LFM.

G.E.A. Franken, et al, Doppler Tolerance of OFDM-Coded Radar Signals, the 3rd European Radar Conference (EuRAD 2006), 13-15 Sept. 2006, Manchester, UK.

Doppler Tolerance of OFDM-Coded Radar Signals

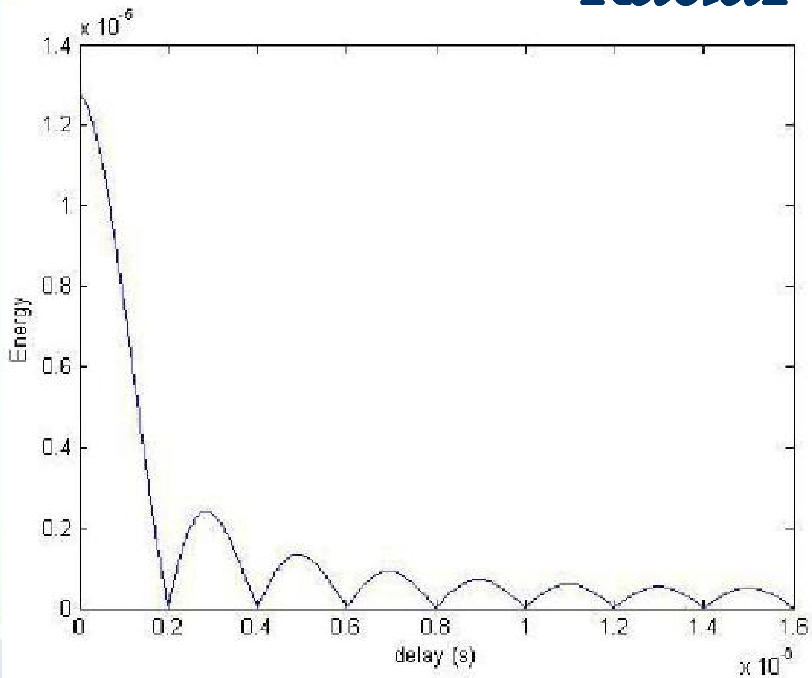
noc	V_{max} (m/s)
8	2,343.75
16	1,171.88
32	585.94
64	292.97
128	146.48
256	73.24
512	36.62
1024	18.31
2048	9.16
4096	4.58



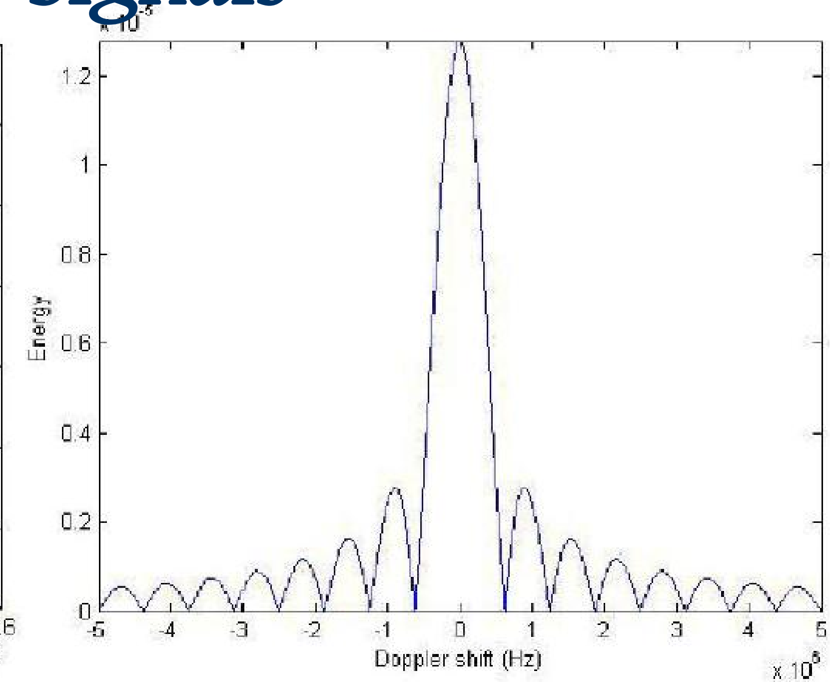
Ambiguity diagram of an eight carrier OFDM signal

The maximum allowed speed for different pulse compression ratio noc, is shown in the table for 1 dB compression loss, when $\mathbf{f}_o$=10GHz, **B**=5MHz.

Doppler Tolerance of OFDM-Coded Radar Signals



Zero-delay cut of the AF of an eight carrier OFDM signal



Zero-Doppler cut of the AF of an eight carrier OFDM signal

- The pulse length after compression is one eighth of the pulse length before compression, achieved without change in the Doppler resolution.
- The compression loss is a function of the Doppler frequency f_d, and the delay.

Single Coded Signals: A Comparison

- Most binary and polyphase pulse compression codes suffer severe signal loss in performance under Doppler environment.
- Frank code is having acceptable amplitude at zero and other selected Doppler values. Having a narrow peak width it is not an ideal Doppler tolerant code.
- P1 & P2 codes are having decent amplitude and wide band for better resolution compared to both frank and barker codes but not ideal for distant targets.
- P3 & P4 codes are having excellent Doppler tolerance in comparison to other codes, good to detect targets at a limited range of speeds.
- HFM code is also excellent in terms of amplitude and Doppler resolution, good in radar applications where the variation in Doppler is very large.
- OFDM-Coded Radar Signals are good for target detection & data tx.

AF Peak of Various RADAR Codes

AF peak reduction for various Doppler shift (f_0=78GHz, v=0-122km/hr)

Doppler shift (Hz)	Velocity (km/h)	16 Bit Frank Code	25 Bit P1/P2 Code	25 Bit P3/P4Code	25 Bit HFM Code
	0.0000	1.0000	1.0000	1.0000	1.0000
400	5.5	0.7551	0.5920	0.9730	0.8901
600	8.3	0.6923	0.4515	0.6652	0.8280
1200	16.6	0.7822	0.6870	0.8572	0.6894
2600	36.0	0.5748	0.4313	0.5878	0.4074
3800	52.6	0.6612	0.5286	0.5398	0.2475
5000	69.2	0.5314	0.3720	0.4734	0.1399
6200	85.8	0.3978	0.4445	0.3869	0.1139
7400	102.5	0.3443	0.3048	0.3014	0.1319
8600	119.1	0.3251	0.2273	0.2695	0.1215

- The AF peak should be high and the peak width be large to give an decent level of amplitude for wide range of Doppler values.
- At 78GHz, P3/P4 codes are excellent in terms of AF peak and Doppler resolution, with the small signal loss over the Doppler shift range.

AF Peak of Various RADAR Codes

AF peak reduction for various Doppler shift (f_0=5.9GHz, v=0-122km/hr)

Doppler Shift (Hz)	Velocity (km/h)	16 Bit Frank Code	25 Bit P1/P2 Code	25 Bit P4 Code	25 Bit HFM Code
0.00	0.00	1.0000	1.0000	1.0000	1.0000
20	3.7	0.9894	0.9379	0.9739	0.9379
60	10.9	0.9072	0.5205	0.7365	0.9229
150	27.5	0.6923	0.4515	0.6652	0.8280
250	45.8	0.8491	0.4552	0.6673	0.7173
350	64.1	0.6336	0.4368	0.6593	0.6200
450	82.4	0.7005	0.5946	0.6421	0.5353
500	91.5	0.7290	0.8970	0.7317	0.5220
600	109.8	0.5762	0.5685	0.6682	0.4528
650	118.9	0.5748	0.4313	0.5878	0.4074

- The AF peak should be high and the peak width be large to give an decent level of amplitude for wide range of Doppler values.
- At 5.9GHz, Frank and HFM codes are excellent in terms of AF peak and Doppler resolution, with the small signal loss over the Doppler shift range.

Golay Complementary Codes

- A key issue in phase coding is the presence of range sidelobes in the ambiguity function of the coded waveforms. Range sidelobes due to a strong reflector can result in masking of nearby weak targets.
- There is no single code sequence with perfect AACF for length >4! Barker codes are the best in terms of AACF, but Doppler sensitive.
- For Golay pair, the two sequences are transmitted alternatively in time over several pulse repetition intervals (PRIs). The effective ambiguity function of a Golay pair of phase coded waveforms is free of range sidelobes along the zero-Doppler axis.

Golay Complementary Pair/Set

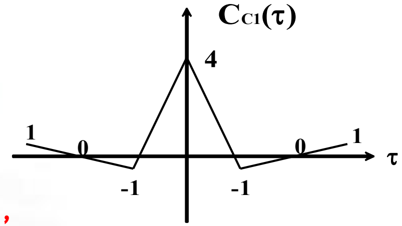
A set of sequences $S_1 = \{s_{1,n}\}$, $S_2 = \{s_{2,n}\}$, …, each of length N, is called complementary (Golay) pair (or set) if

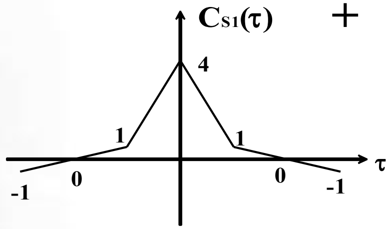
$$\text{Golay Pair}: C_{S_1}(\tau) + C_{S_2}(\tau) = 0, \quad \tau \neq 0$$

$$\text{Golay Set}: \sum_{p=1}^{P} C_{S_p}(\tau) = 0, \quad \tau \neq 0$$

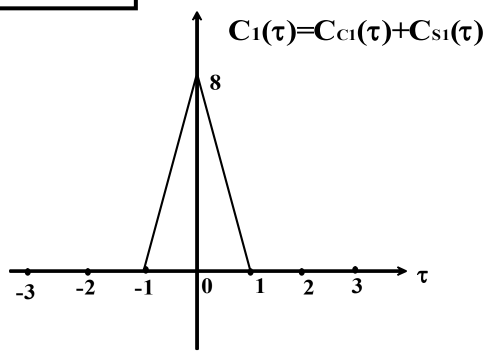
$$\text{where} \quad C_{S_i}(\tau) = \sum_{n=0}^{N-\tau-1} s_{i,n} s^{*}_{i,n+\tau}$$

- Binary Golay pairs exist only for lengths $N=2^a 10^b 26^c$ for a,b,c≥0, i.e. 2, 4, 8, 10, 16, 20, 26, 32, 52, 64, 128, …
- Quadriphase Golay pairs exist only for lengths N=2, 3, 4, 5, 6, 8, 10, 12, 13, 14, 16, 18 and so on.





+ =

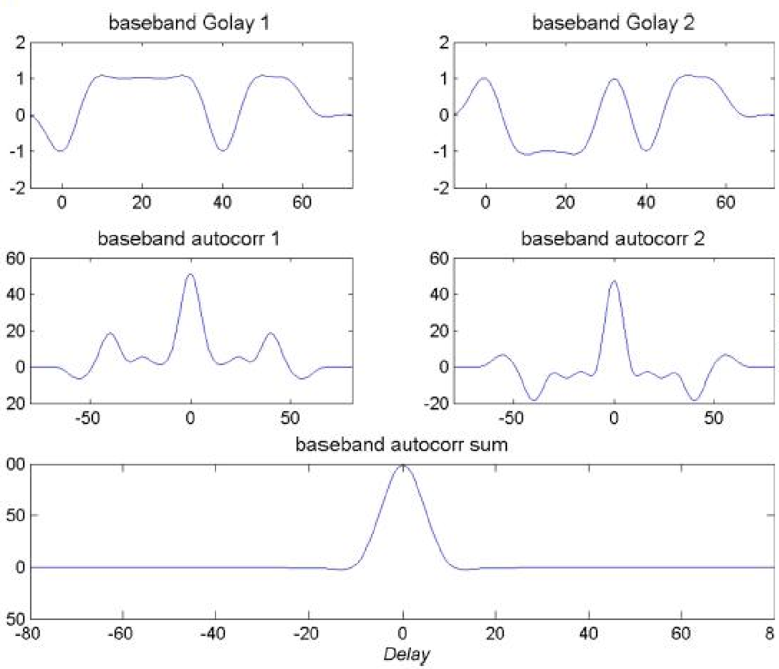


A Golay Pair:
(C,S)= (++−+, +−−−)

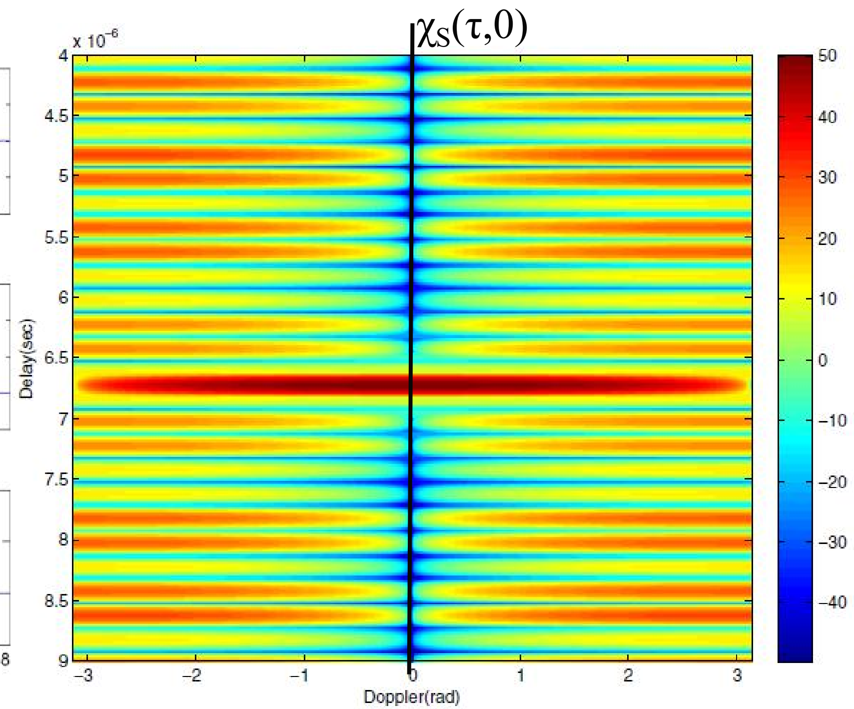
Ambiguity Function of Golay Codes

$$S(t)=s_x(t)+s_y(t-T),\quad s_x(t)=\sum_{k=1}^{N-1}x_k P_{T_C}(t-kT_C),\quad P_{T_C}(t)=\begin{cases}1, & 0\le t<T_C\\ 0, & otherwise\end{cases}$$

$$\chi_{S_x}(\tau,\nu)=\int_{-\infty}^{\infty}s_x(t)x_x^*(t-\tau)e^{-j\nu t}dt \xrightarrow{NT_C<<T} \chi_S(\tau,\nu)=\sum_{k=-(N-1)}^{N-1}\left[C_x(k)+e^{j\nu T}C_y(k)\right]\chi_{P_{T_C}}(\tau+kT_C,\nu)$$



Perfect Aperiodic ACF of a Golay pair

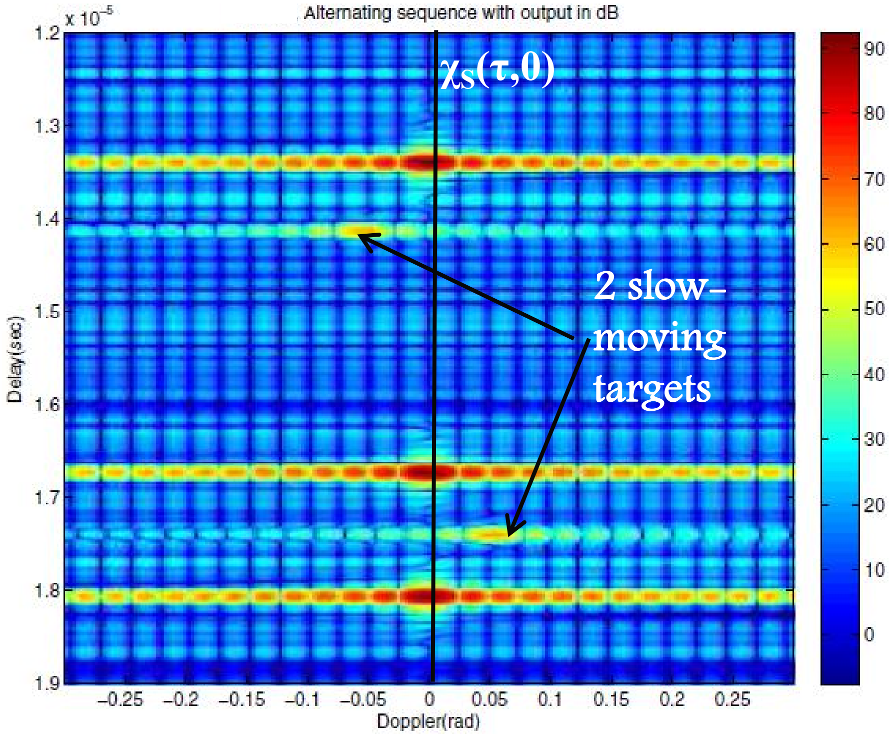


Ambiguity function of a Golay pair

Doppler Resilient Requirement

- However, the ideal aperiodic ACF property of Golay codes is sensitive to Doppler effect.
- Off the zero-Doppler axis the ambiguity function of Golay pairs of phase coded waveforms has **large range sidelobes**, a major barrier for radar pulse compression.
- Is it possible to construct a Doppler resilient pulse train of Golay complementary waveforms, for which the range sidelobes of the ambiguity function vanish inside a desired Doppler interval?

256 PRIs (T=50μs), N=64, Tc=100ns, f_0=17GHz



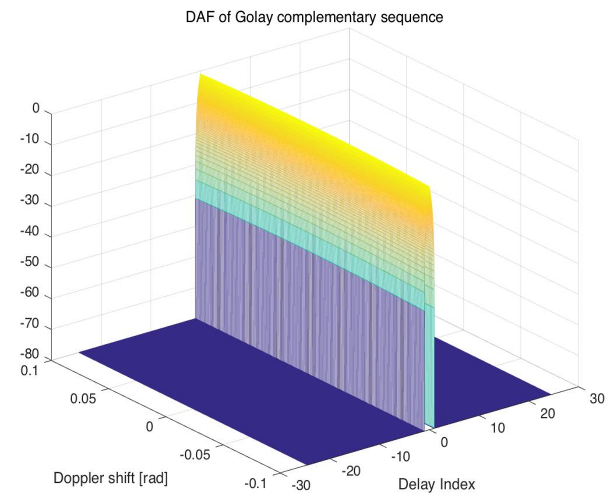
This radar scene contains 3 stationary reflectors at different ranges and 2 slow-moving targets, which are 30dB weaker than the stationary reflectors

Outline

- Automotive Radars and Related Signals
- Pulse Compression and Phase Coding
- Doppler Resilient Sequences (DRS)
- DRS Design based on Z-Ambiguity
- Seqs for Optimized AF & PAPR in CR

Doppler Resilient Sequences (DRS)

- Pezeshki et al showed, by carefully choosing the order in which a Golay pair of phase coded waveforms $s_x(t)$ and $s_y(t)$ is transmitted over time one can clear out the range sidelobes of the pulse train ambiguity function along modest (close to zero) Doppler shifts.
- If the transmission of a Golay pair of phase coded waveforms is coordinated in time according to the entries in a biphase sequence, then the magnitude of the range sidelobes can be controlled by shaping the spectrum of the biphase sequence.



[1] A. Pezeshki, A. R. Calderbank, W. Moran, and S. D. Howard, "Doppler resilient Golay complementary waveforms," IEEE Trans. Inform. Theory, vol. 54, no. 9, Sept. 2008.
[2] A. Pezeshki, A. R. Calderbank, et al, "Doppler resilient Golay complementary pairs for radar," in Proc. Stat. Signal Proc. Workshop, Madison, WI, Aug. 2007, pp. 483–487.
[3] Yuejie Chi, Ali Pezeshki, et al, "Range sidelobe suppression in a de- sired Doppler interval," IEEE International Waveform Diversity and Design Conference, 258-262, 2009.

Doppler Resilient Golay Pulse Train

Consider a biphase sequence P={p_n}, $p_n \in \{-1,1\}$, $0 \le n \le L-1$, length L is even. Let p_n=1 and −1 represent $s_x(t)$ and $s_y(t)$ respectively, (x,y) is a Golay pair. Then a **P-pulse train $S_P(t)$** of ($s_x(t)$, $s_y(t)$) is defined as

$$S_P(t) = \frac{1}{2}\sum_{n=0}^{L-1}\left[(1+p_n)s_x(t-nT)+(1-p_n)s_y(t-nT)\right]$$

The *n*th entry in $S_P(t)$ is $s_x(t)$ if p_n=1, and $s_y(t)$ if p_n=−1. Consecutive entries are separated in time by a pulse repetition interval (PRI) T. The **ambiguity function** (AF) of $S_P(t)$, after ignoring the pulse shape AF, discretizing in delay, relative Doppler shift $\theta = \nu T$, is

range sidelobes

$$\chi_{S_P}(k,\theta) = \frac{1}{2}\left[C_x(k)+C_y(k)\right]\sum_{n=0}^{L-1} e^{jn\theta} + \frac{1}{2}\left[C_x(k)-C_y(k)\right]\sum_{n=0}^{L-1} p_n e^{jn\theta}$$

zero for k<N in Golay pair

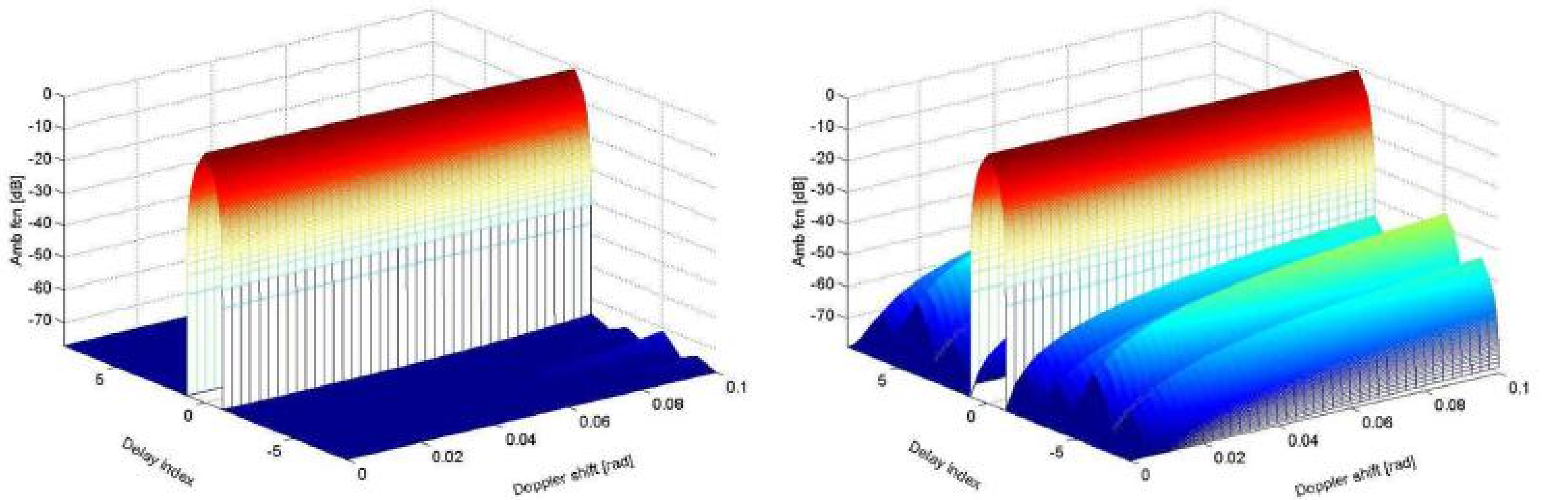
the spectrum of PTM {p_n} of length 2^{M+1} has an M^{th}-order null at θ=0

Prouhet-Thue-Morse (PTM) Sequence

- PTM sequence was introduced by Thue in 1906 and rediscovered by Morse in 1921, but was implicit in an 1851 paper of Prouhet.
- Denote by $P=\{p_n\}$, $n\geq 0$, the Prouhet-Thue-Morse (PTM) sequence over $\{-1,1\}$, **defined recursively by $p_0=1$, and $p_{2n}=p_n$, $p_{2n+1}=-p_n$.**
- The spectrum of PTM $\{p_n\}$ of length 2^{M+1} has an M^{th}-order null at $\theta=0$.
- **Example**: The PTM sequence of length $L=2^{2+1}$ is $P=(+1\ -1\ -1\ +1\ -1\ +1\ +1\ -1)$, and the PTM pulse train of Golay complementary waveforms is $S_P(t)=s_x(t)+s_y(t-T)+s_y(t-2T)+s_x(t-3T)+s_y(t-4T)+s_x(t-5T)+s_x(t-6T)+x_y(t-7T)$, the AF of $S_P(t)$ has a 2^{nd}-order null along the zero-Doppler axis.

Doppler Resilient PTM Pulse Train

16 PRIs (T=50μs), N=8, Tc=100ns, f_0=17GHz



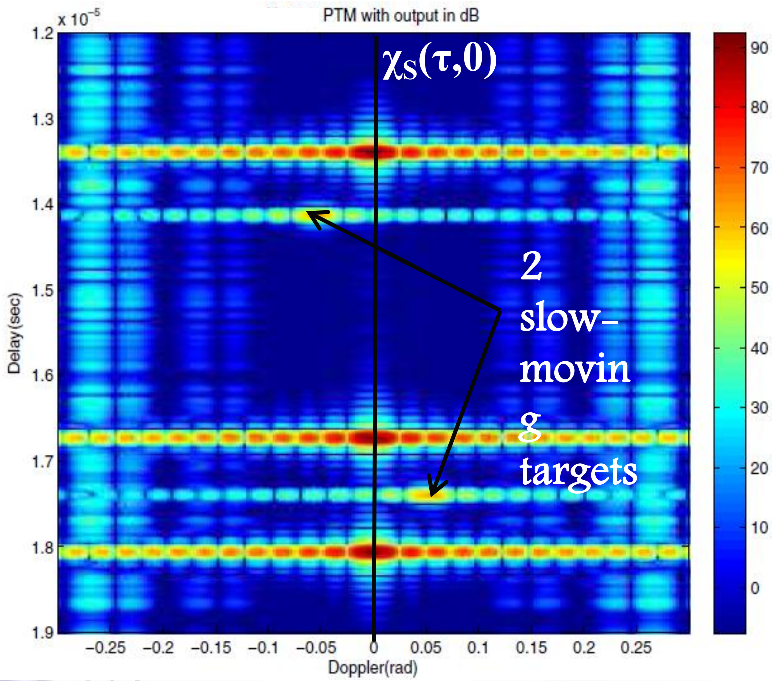
(a) $g(\ell,\theta)$ (b) $g_c(\ell,\theta)$

Doppler resilient transmission scheme | Conventional transmission scheme

Ambiguity function of a length $L=2^{3+1}$ PTM pulse train of Golay complementary waveforms, which has a 3^{th}-order null at zero-Doppler

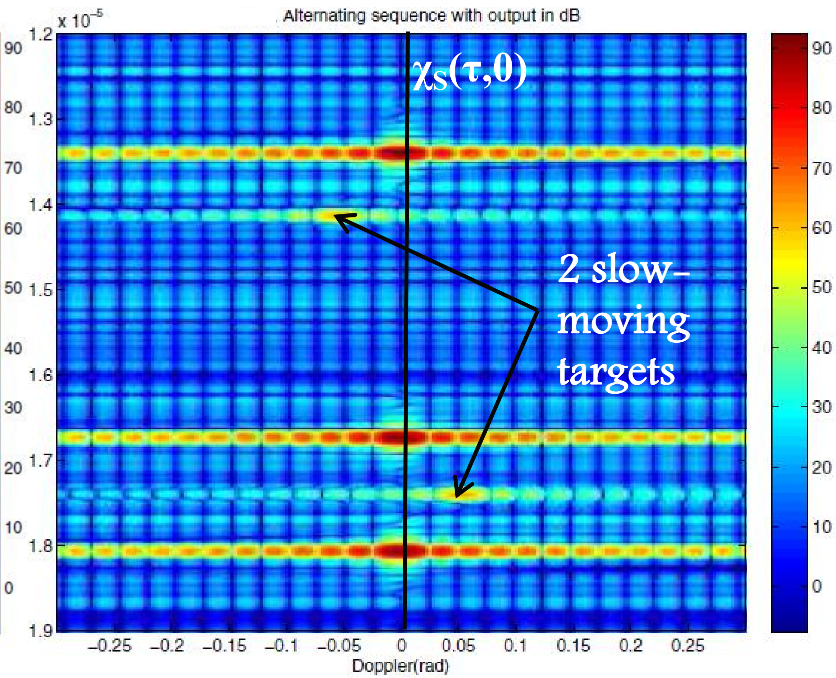
A. Pezeshki, A. R. Calderbank, W. Moran, and S. D. Howard, "Doppler resilient Golay complementary waveforms," *IEEE Trans. Inform. Theory*, vol. 54, no. 9, Sept. 2008.

A Radar Scene with Alternating/PTM Sequences (5 Targets, 2 Slow Moving)



PTM Sequence

(256 PRIs (T=50μs), N=64, Tc=100ns, f_0=17GHz)



Alternating Sequence

(256 PRIs (T=50μs), N=64, Tc=100ns, f_0=17GHz)

Prouhet-Tarry-Escott Problem

Prouhet-Tarry-Escott Problem: Let A and B be two disjoint subsets of *n* integers each. Then A, B are equal sums of (like) powers (ESP) sets of degree k if

$$\sum_{n\in A} a^i = \sum_{n\in B} b^i$$

for i=1,...,**k**. Solutions with **k=n-1** are called **ideal solutions**, existing for 3≤n≤10 & n=12. No ideal solution is known for n=11 or for n≥13.

Example: Since A={0,4,5} and B={1,2,6} in P=A∪B={0,1,2,4,5,6} form an ESP pair of degree k=2 (n=3, ideal), it is obvious that the pair A'={0,3,4,5} and B'={1,2,36} in P'=A'∪B'={0,1,2,3,4,5,6} is also an **ESP pair** of degree k=2 (n=4, non-ideal).

$$0^i + 4^i + 5^i = 1^i + 2^i + 6^i, \quad i = 1,2$$

$$0^i + \boxed{3^i} + 4^i + 5^i = 1^i + 2^i + \boxed{3^i} + 6^i, \quad i = 1,2$$

PTM Sequence and ESP Sequence

Prouhet-Thue-Morse (PTM) Sequence is a special case of ESP sequence with $n=2^k$, namely, partition the numbers from 0 to $2^{k+1}-1$ into the evil numbers and the odious numbers, thus giving a non-ideal solution ($k \neq n-1$) to the Prouhet-Tarry-Escott Problem.

Example: For PTM Sequence of $n=2^{3+1}=8$, $k=3$, Prouhet's solution is: A={0, 3, 5, 6, 9, 10, 12, 15}, B={1, 2, 4, 7, 8, 11, 13, 14}, satisfying

$0^i + 3^i + 5^i + 6^i + 9^i + 10^i + 12^i + 15^i = 1^i + 2^i + 4^i + 7^i + 8^i + 11^i + 13^i + 14^i$, for i=1,2,3.

PTM sequence: (0 1 1 0 1 0 0 1 1 0 0 1 0 1 1 0 1 0 0 1 0 1 1 0···)
or bipolar form: (+1 −1 −1 +1 −1 +1 +1 −1 −1+1 +1 −1 +1 −1 −1 ···)

Example: An ideal ESP (non-PTM) sequence of n = 6, k=n−1=5, the two sets are: A={ 0, 5, 6, 16, 17, 22 } and B={ 1, 2, 10, 12, 20, 21 }.

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Doppler Resilient ESP Pulse Train

- The characterisation of **Doppler-null codes** bears a striking resemblance to the characterisation of **spectral-null codes**.
- It can be shown that Doppler-null codes have higher-order zeros, it follows from a well-known result in calculus that the higher order derivatives in their ambiguity functions must vanish.
- ESP pulse trains provide the same Doppler tolerance as PTM pulse trains, but are generally shorter in length, by using multiple antennas to transmit separate pulse trains staggered in time.

Example: As shown earlier, A={0,4,5} and B={1,2,6} in $P=A\cup B=\{0,1,2,4,5,6\}$ form an ESP pair of degree k=2 (n=3, ideal). Giving Golay pair (x, y), we can form a pulse train **T=(x, y, y, x+y, x, x, y)** with a gap at position 3, A'={0,**3**,4,5} and B'={1,2,**3**,6} is also an ESP pair of degree k=2 (n=4).

H D Nguyen1, G E Coxson, Doppler tolerance, complementary code sets, and generalised Thue–Morse sequences, **IET Radar Sonar Navig**., 2016, Vol. 10 Iss. 9, pp. 1603-1610.

Doppler Resilient ESP Pulse Train

Then, instead of single pulse train **T=(x, y, y, x+y, x, x, y),** one can transmit two separate pulse trains of length 4 with Golay pair (x, y), i.e. T_0, and 3 PRIs delayed T_1

T_0 = (**x, y, y, x**) % tx from antenna No.1

$T_1(3)$= (**y, x, x, y**) % tx from antenna No.2, delayed by 3 PRIs

To show the AF g(k,) has Doppler nulls of order 2 at =0, one can compute its Doppler (Taylor) coefficients,

$$c_i(k)=g^{(i)}(k,0)=(0^i+3^i+4^i+5^i)C_x(k) + (1^i+2^i+3^i+6^i)C_y(k)$$

$$=P_i\,(C_x(k)+C_y(k))=2NP_i\,\delta_k, \qquad for\ i=0,1,2$$

Thus achieving the same Doppler tolerance as with a single PTM pulse train of length 8 by using instead two staggered (but overlapping) pulse trains of length 4, although the total number of pulses transmitted is the same, namely 8, in both cases.

Doppler Resilient ESP Pulse Train

Example: An ESP pair of n=4, k=3, A={0, 4, 7, 11}, B={1, 2, 9, 10},

$$0^i + 4^i + 7^i + 11^i = 1^i + 2^i + 9^i + 10^i, \text{ for } i=1,2,3$$

A'={0, 3, 4, 5, 6, 7, 8, 11}, B'={1, 2, 3, 5, 6, 8, 9, 10} is also an ESP pair of degree k=3 (n=8)

We now transmit 4 pulse trains T_0, $T_1(3)$, $T_2(5)$, $T_3(8)$ on 4 separate antennas having delays 0, 3, 5, 8, respectively.
The total transmission time from 16 pulses (for a single PTM pulse train of length 16 having the same Doppler tolerance) is reduced down to 12 by using instead 4 pulse trains transmitted separately, although the total number of pulses transmitted is the same (16) in both cases.

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- Automotive Radars and Related Signals
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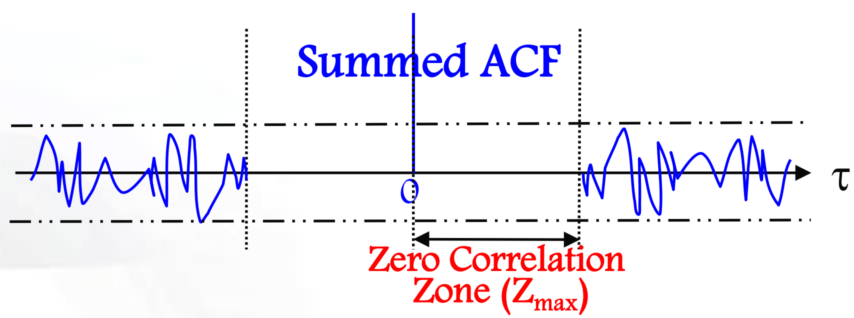
Z-Complementary Sequences

A set of P sequences $\{S_1, S_2, \ldots S_P\}$, each having length N, is called a set of Z-complementary sequences (Z-$CS_{P,N}$) if

$$\sum_{p=1}^{P} C_{S_p}(\tau) = \begin{cases} NP, & \tau = 0 \\ 0, & 1 \le \tau \le Z_{\max} - 1 \end{cases} \qquad C_{S_i}(\tau) = \sum_{n=0}^{N-\tau-1} s_{i,n} s_{i,n+\tau}$$

The Z-complementary pair of binary or quadriphase sequences exists essentially **for all lengths** for different $Z_{max} \le N$.



[1] Pingzhi Fan, Weina Yuan, et al, Z-complementary binary sequences, **IEEE Signal Processing Letters**, Vol. 14, No.8, August 2007, pp.509-512.
[2] Lifang Feng, Pingzhi Fan, et al, Generalized Pairwise Z-complementary Codes, **IEEE Signal Processing Letters**, Vol.15, pp.377-380, 2008.

Kernels of Binary Z–Golay Pairs

N	Z_m	Example of Z-complementary sets (C,S)	Summed ACF: $A_C(k)+A_S(k)$
2	2	(++; +-)	(4, 0)
3	2	(+++， +-+)	(6, 0, 2)
5	3	(++++-， +-++-)	(10, 0, 0, 2, -2)
7	4	(++++--+， ++-+-++)	(14, 0, 0, 0, -2, 2, 2)
9	5	(+++++--++， ++-+-+--+)	(18, 0, 0, 0, 0, 2, -2, 2, 2)
10	10	(+--+ - +---+; +------++-)	(20, 0, 0, 0, 0, 0, 0, 0, 0, 0)
11	6	(+++++-+-+--, ++---++-++-)	(22, 0, 0, 0, 0, 0, -2, -2, 2, -2, -2)
12	10	(++++--+-+-++， +-++++++--++-)	(24, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0)
13	7	(++++++--+-+-+， +++--+-++--++)	(26, 0, 0, 0, 0, 0, 0, 2, -2, -2, 2, 2, 2)
14	12	(+++++-+--+++--， +-++-+---+---+)	(28, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -4, 0)
15	8	(++++++-+--++++-， +-+--++--+++-+-)	(30, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 6, -2, 2, -2)
17	9	(-------+-+++---++,--+-++--++-+-+--+)	(34, 0, 0, 0, 0, 0, 0, 0, 0, 2, -6, 2, 2, -2, 2, -2,-2)
18	13	(----+--+--+---++++,-+++---+-+-++-+++-)	(36,0,0,0,0,0,0,0,0,0,0,0,0,4,-4,-4,-4,0)
19	10	(+--------++--+-++--,+--+++-+----++-+-+-)	(38, 0, 0, 0, 0, 0, 0, 0, 0, 0, 6, -6, 2, 2, -2, 2, 2, 2, -2)
21	11	(+--------+-++-+--+++-,+-+---+++--++-+++-+--)	(42, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -6, 2, -6, 2, -2, 2, -2, 2, 2, -2)
22	17	(--------+++-+-+--++-++,-+--++----+++--+-+-++-)	(44,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-8,0,0,-4,0)
23	12	(-------+-+--+++-++----+,--+-+--++--+-+++---+-++)	(46,0,0,0,0,0,0,0,0,0,0,0,0,-6,-2,-2,6,-2,6,2,2,2,-2,-2)
25	13	(--------+-++-+---++-++--+,-++-+-+---+-++++--+++---+)	(50,0,0,0,0,0,0,0,0,0,0,0,0,0,2,6,-6,2,-2,2, -2,-2,-2,2,2,-2)
26	26	(+ + +--+++- +-----+ -++--+----; ---++---+ -++- + - + -++--+----)	(52,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

Kernels of Quadriphase Z-Golay Pairs

N	Z_{max}	Example of generators	Summed ACF
3	3	(1,1,-1; 1, i, 1)	(6, 0, 0)
4	4	(1,1,1,-1; 1,1,-1,1)	(8, 0, 0, 0)
5	5	(1,1,1,–i, i ; 1, i, –i,1,i)	(10, 0, 0, 0, 0)
6	6	(1,1,1,i ,-1,1; 1,1,–i ,-1,1,-1)	(12, 0, 0, 0, 0, 0)
7	6	(1,1,1,1,-1,-1,1; 1, i, –i ,1, –i ,i ,1)	(14, 0, 0, 0, 0, 0, 2)
8	8	(1,1,1,1,1,-1,-1 1; 1,1,-1,-1,1,-1,1,-1)	(16, 0, 0, 0, 0, 0, 0, 0)
9	8	(1,-1,i ,1, i ,–i ,–i ,–i ,1; 1,1,1,i ,–i ,1, -1, i ,1)	(18, 0, 0, 0, 0, 0, 0, 0, 2)

[1] X. D. Li, P. Z. Fan, Constructions of Quadriphase Z-complementary Sequences, **IWSDA'2009**, October, 2009, Fukuoka, Japan.
[2] X. Li, P. Fan, et al, Existence of binary Z-complementary pairs," **IEEE SPL**, vol.18, no.1, 2011.
[3] Zilong Liu, Udaya Parampalli, Yong Liang Guan, Optimal Odd-Length Binary Z-Complementary Pairs, **IEEE Trans on Information Theory**, 2014, Vol.60, No.9.
[4] X. Li, et al, New construction of Z-complementary pairs, **Electron. Lett.**, vol.52, no.8, 2016.
[5] Chao-Yu Chen, A Novel Construction of Z-Complementary Pairs Based on Generalized Boolean Functions, **IEEE SPL**, Vol.24, NO.7, JULY 2017.

Doppler Resilient Z-Golay Pulse Train

Consider a biphase sequence P={p_n}, $p_n \in \{-1,1\}$, $0 \le n \le L-1$, length L is even. Let p_n=1 and -1 represent $s_x(t)$ and $s_y(t)$ respectively, (x,y) is a Z-Golay pair. Then a **P-pulse train $S_P(t)$** of ($s_x(t)$, $s_y(t)$) is defined as

$$S_P(t) = \frac{1}{2}\sum_{n=0}^{L-1}\left[(1+p_n)s_x(t-nT)+(1-p_n)s_y(t-nT)\right] \qquad s_x(t) = \sum_{k=1}^{N} x_k P_{T_C}(t-kT_C)$$

The *n*th entry in $S_P(t)$ is $s_x(t)$ if p_n=1, and $s_y(t)$ if p_n=-1. Consecutive entries are separated in time by a PRI, i.e. *T*. The **ambiguity function** (AF) of $S_P(t)$, after ignoring the pulse shape AF, discretizing in delay, relative Doppler shift $\theta = \nu T$, is

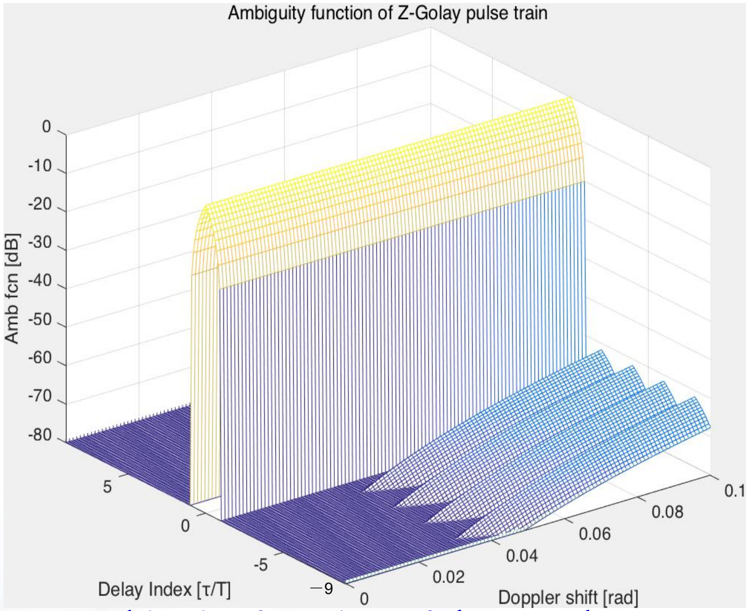
range sidelobes

$$\chi_{S_P}(k,\theta) = \frac{1}{2}\left[C_x(k)+C_y(k)\right]\sum_{n=0}^{L-1} e^{jn\theta} + \frac{1}{2}\left[C_x(k)-C_y(k)\right]\sum_{n=0}^{L-1} p_n e^{jn\theta}$$

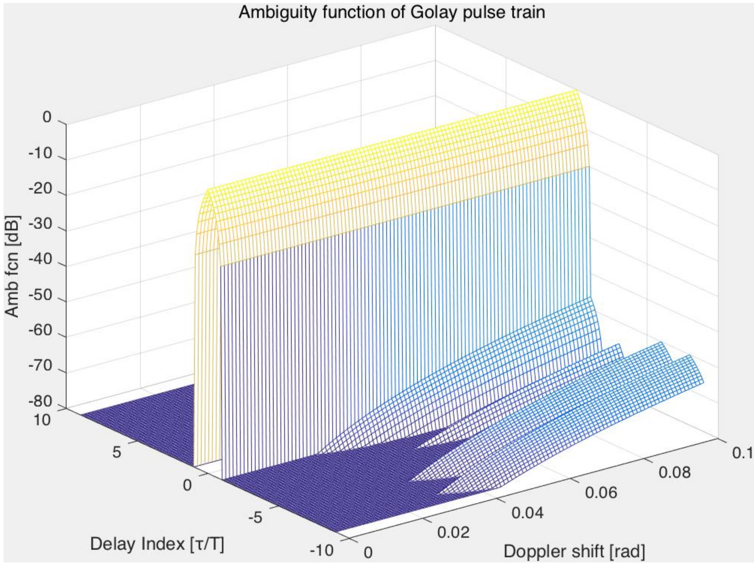
zero for k<Z_m in Z-Golay pair

the spectrum of PTM {p_n} of length 2^{M+1} has an M^{th}-order null at θ=0

Doppler Resilient Z-Golay Pulse Train



Ambiguity function of the 1 order PTM pulse train of Z-Golay (N=12,Z_m=10).

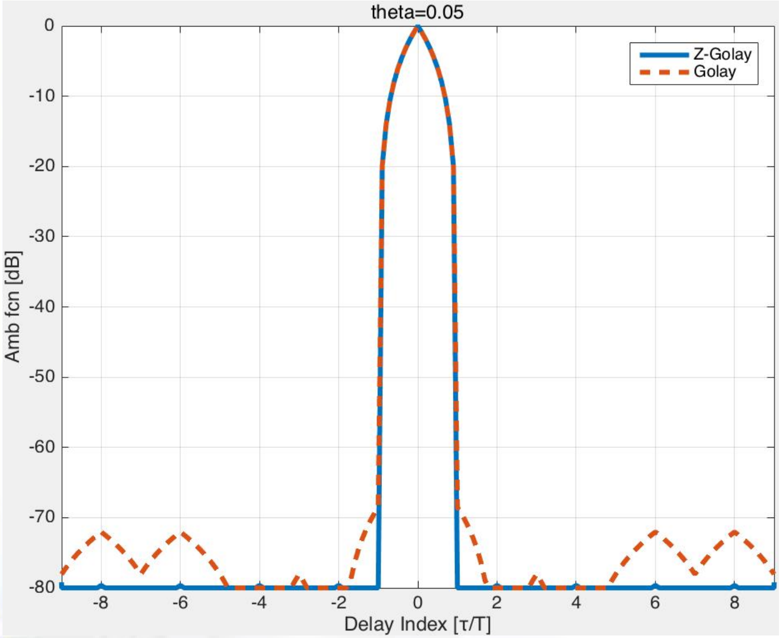


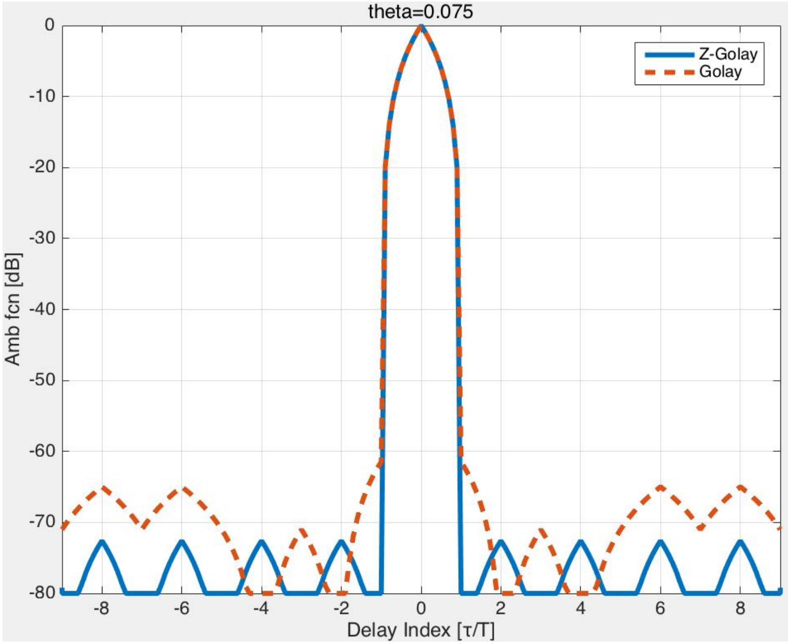
Ambiguity function of the 1 order PTM pulse train of Golay (N=10).

Z-Golay(N=12,Z_m=10): (x,y)=(1,1,1,1,-1,-1,1,-1,1,-1,1,1; 1,-1,1,1,1,1,1,-1,-1,1,1,-1);
Golay Code(N=10): (x,y)=(1,-1,-1,1,-1,1,-1,-1,-1,1; 1,-1,-1,-1,-1,-1,-1,1,1,-1)
PTM: (1 -1 -1 1): transmission pulse train {x y y x}

Due to the **volume invariance** property of AF, Z-Golay codes behaves better than Golay code within zero correlation zone Z_m, due to the bigger AF values outside the zero correlation zone in Z-Golay codes

Doppler Resilient Z-Golay Pulse Train



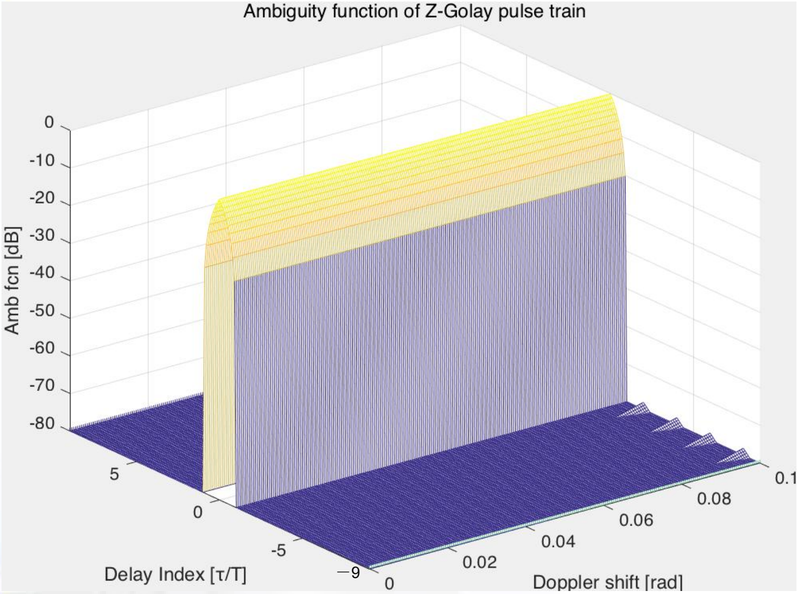


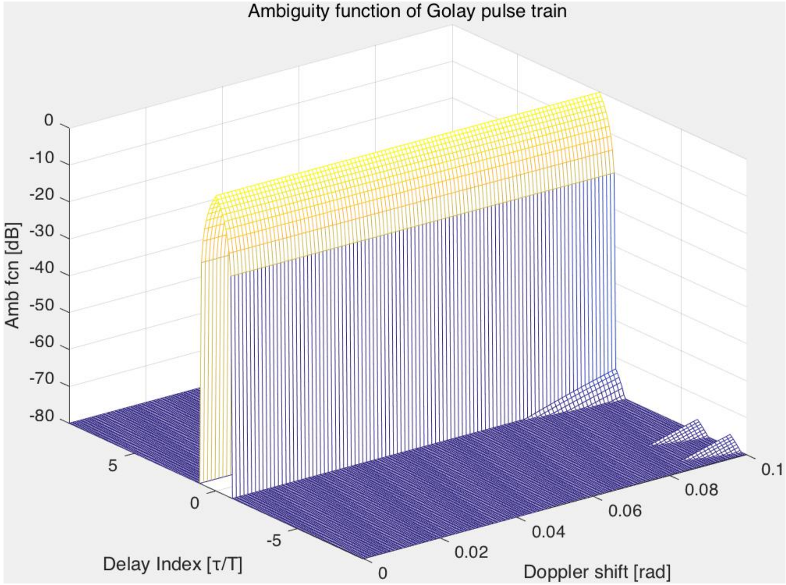
Ambiguity function of the 1st order PTM pulse train of Z-Golay (N=12, Z_m=10) and Golay waveform(N=10), at θ=0.05 and 0.075 respectively

Z-Golay(N=12,Z_m=10): (x,y)=(1,1,1,1,-1,-1,1,-1,1,-1,1,1; 1,-1,1,1,1,1,1,-1,-1,1,1,-1);
Golay Code(N=10): (x,y)=(1,-1,-1,1,-1,1,-1,-1,-1,1; 1,-1,-1,-1,-1,-1,-1,1,1,-1)
PTM: (1 -1 -1 1): transmission pulse train: {x y y x}

In many application, Z-Golay with Z_m (zero correlation zone) is enough.

Doppler Resilient Z-Golay Pulse Train



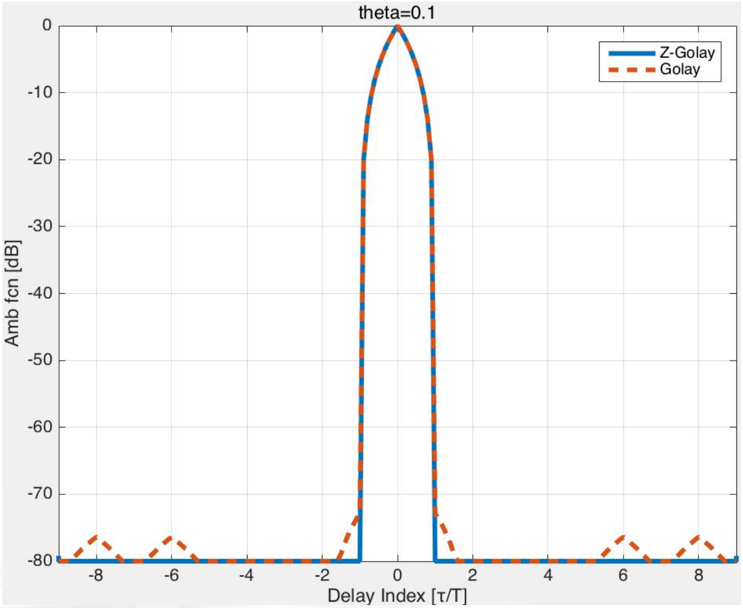


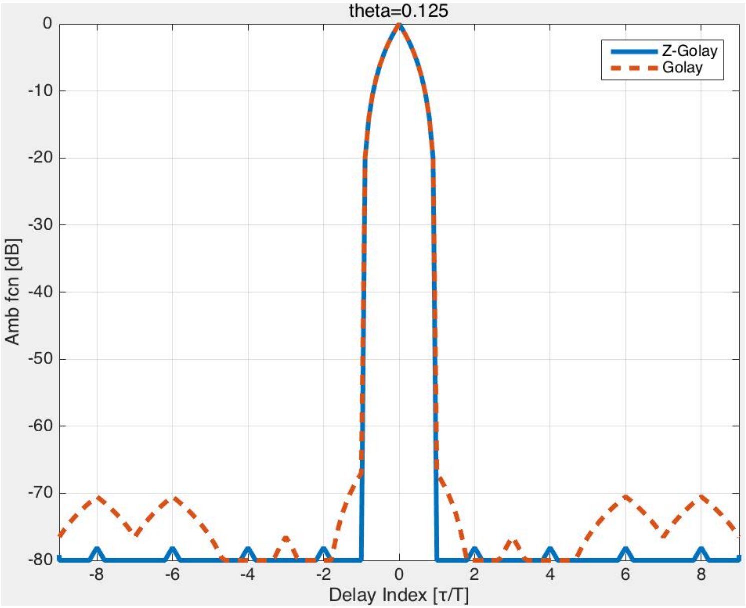
Ambiguity function of the degree=2 ESP pulse train of Z-Golay (N=12,Z_m=10)

Ambiguity function of the degree=2 ESP pulse train of Golay (N=10)

Z-Golay(N=12,Z_m=10): (x,y)=(1,1,1,1,-1,-1,1,-1,1,-1,1,1; 1,-1,1,1,1,1,1,-1,-1,1,1,-1);
Golay Code(N=10): (x,y)=(1,-1,-1,1,-1,1,-1,-1,-1,1; 1,-1,-1,-1,-1,-1,-1,1,1,-1)
ESP: (0 4 5),(1 2 6): transmission pulse train {x y y x+y x x y}

Doppler Resilient Z-Golay Pulse Train





Ambiguity function of the degree=2 ESP pulse train of Z-Golay ($N=12, Z_m=10$) and Golay waveform ($N=10$) at $\theta=0.1$ and 0.125 respectively

Z-Golay($N=12, Z_m=10$): (x,y)=(1,1,1,1,−1,−1,1,−1,1,−1,1,1; 1,−1,1,1,1,1,1,−1,−1,1,1,−1);
Golay Code(N=10): (x,y)=(1,−1,−1,1,−1,1,−1,−1,−1,1; 1,−1,−1,−1,−1,−1,−1,1,1,−1)
ESP: (0 4 5),(1 2 6): transmission pulse train {x y y x+y x x y}

ESP Doppler Tolerance at θ=2πi/m

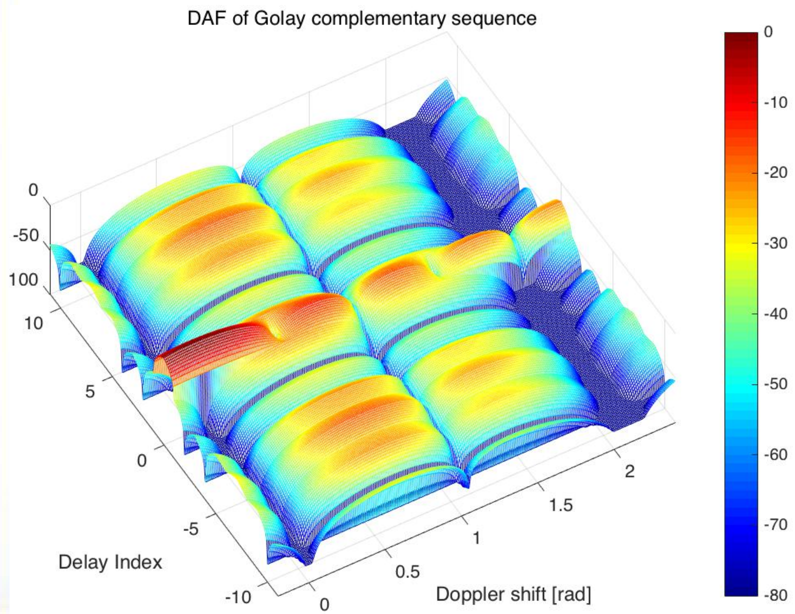
- In addition to Doppler tolerance in the neighborhood of zero Doppler shift, Chi et al considered the rational Doppler shift in neighborhood of rational θ=2π i /m (away from zero Doppler) for which PTM sequences are still important.
- The idea is to oversample the PTM sequence by a factor m.
- This idea can also be applied to **ESP** sequences with **Z-Golay** pulse train to achieve Doppler resilience in the neighborhood of rational θ=2π i /m.

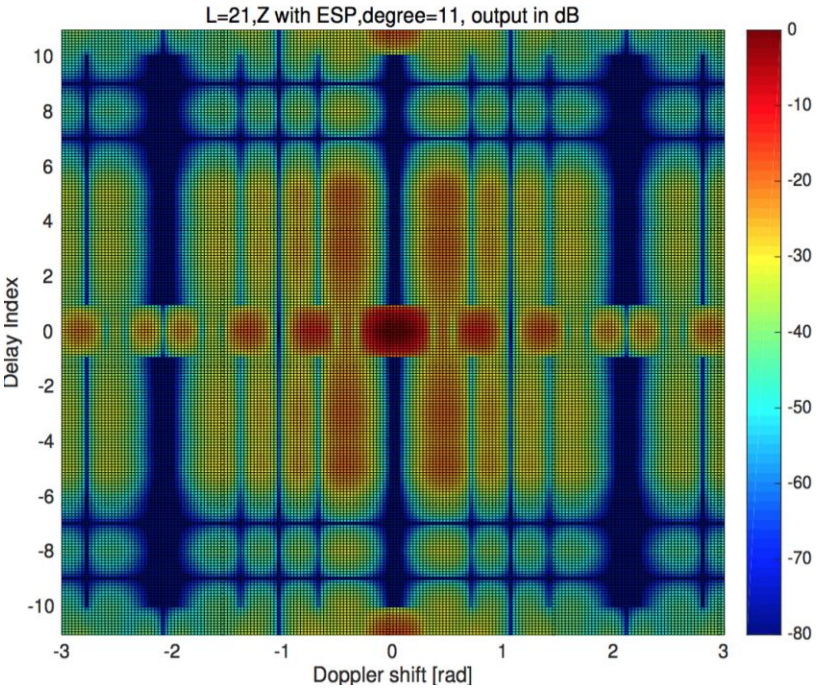
Example: For ESP pulse trains: A'={0,3,4,5}, B'={1,2,3,6} -> (x, y, y, x+y, x, x, y), by oversampling with factor m=3, i.e. x ->xxx, y->yyy, and delayed 9 PRIs

T0= (x, y, y, **x**), -> T0'= (xxx, yyy, yyy, **xxx**)
T1(3)= (**y**, x, x, y), -> T1'(9)= (**yyy**, xxx, xxx, yy)

Yuejie Chi, Ali Pezeshki, Robert Calderbank, et al, "Range sidelobe suppression in a de- sired Doppler interval," *IEEE Int Waveform Diversity and Design Conf*, 2009.

Doppler Resilient Z-Golay Pulse Train





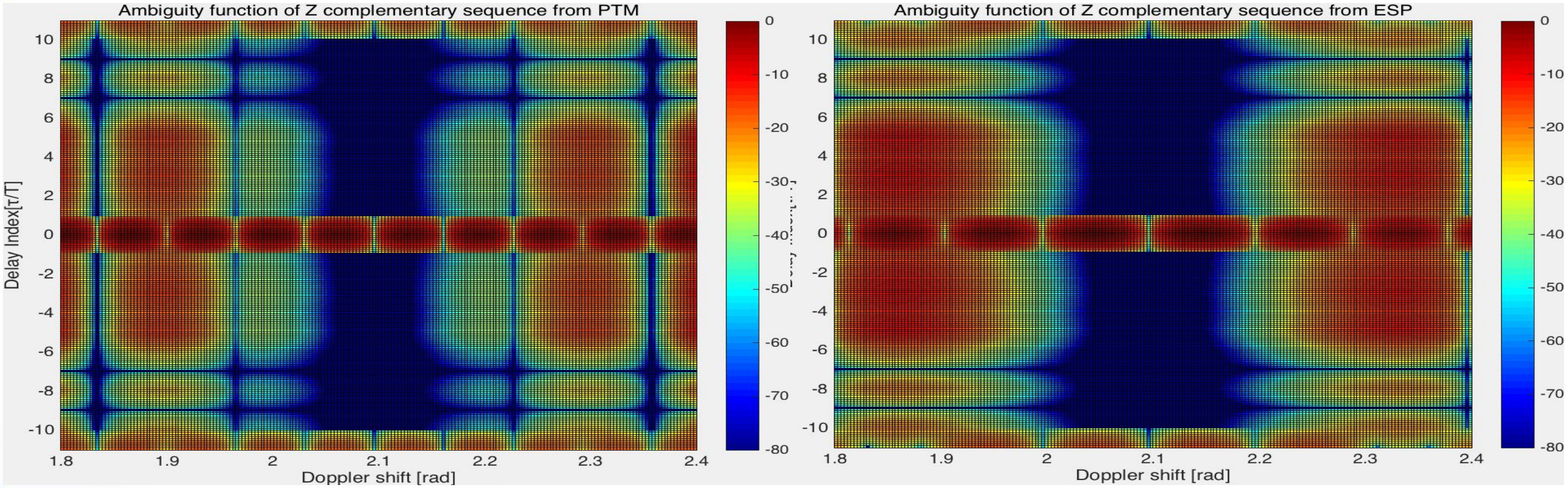
Ambiguity function of the degree=2 ESP pulse train of Z-Golay waveform, N=21, Z_m=11

x = [1 -1 -1 -1 -1 -1 -1 -1 -1 1 -1 1 1 -1 1 -1 -1 1 1 1 -1];

y = [1 -1 1 -1 -1 -1 1 1 1 -1 -1 1 1 -1 1 1 1 -1 1 -1 -1];

JiahuanWANG, Pingzhi FAN, et al, Doppler Resilient Z-complementary Waveforms From ESP Sequences, *the 8th Int. Workshop on Signal Design and its Applications in Communications (IWSDA'17)*, 24-28 Sept, 2017, Sapporo, Japan.

ESP versus PTM Z-Golay Pulse Train

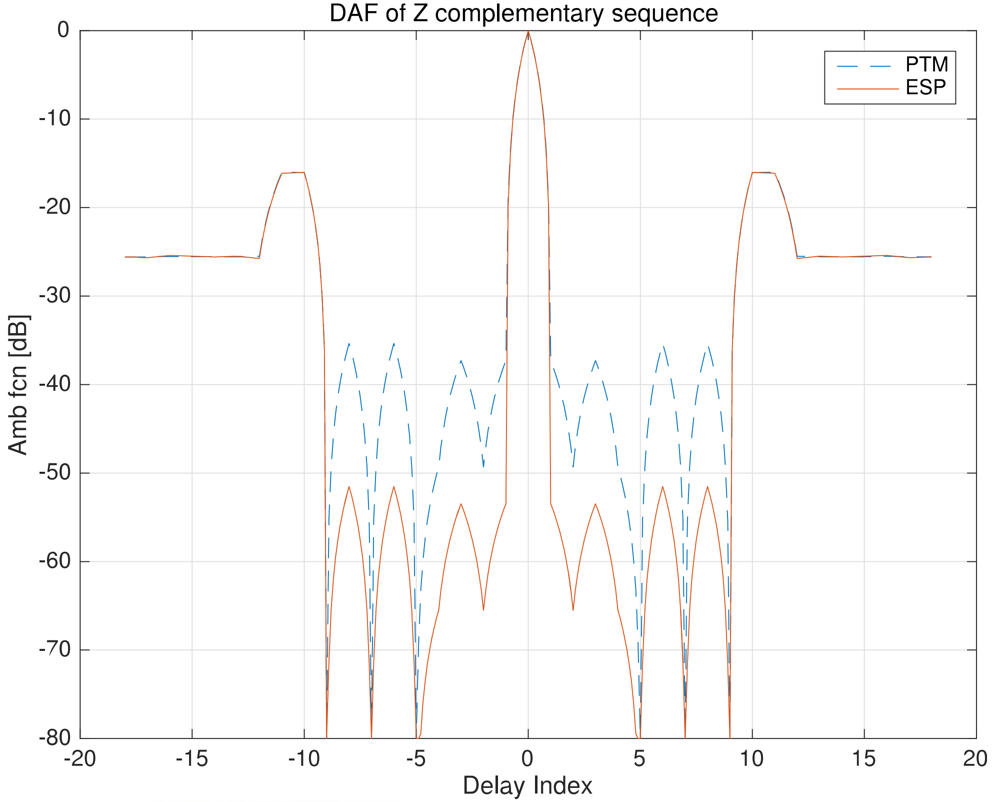


AF of order 5 **PTM** pulse train of Z-Golay waveform at θ=2π/3

AF of the degree 5 **ESP** pulse train of Z-Golay waveform at θ=2π/3

ESP's Doppler tolerant performance at θ=2π/3 is noticeably better than PTM's, as the band around θ=2π/3 is clearly broader in ESP Z-Golay case.

ESP versus PTM Z-Golay Pulse Train



A cut of ambiguity function of the order 5 PTM and degree 5 ESP at Doppler shift $\theta = 2\pi/3 - 0.075$ rad.

In proper delay interval [−9, 9], the peaks of sidelobe determined by ESP are at least 15dB smaller than those determined by PTM.

Outline

- Automotive Radars and Related Signals
- Pulse Compression and Phase Coding
- Doppler Resilient Sequences (DRS)
- DRS Design based on Z-Ambiguity
- Seqs for Optimized AF & PAPR in CR

Seqs for Optimized AF & PAPR in CR

- To solve the scarcity of available spectrum, **cognitive radio (CR)** is proposed to provide the capability of using and sharing spectrum in an opportunistic manner.
- The spectrum opportunity is defined as **spectrum holes** which are not being used by the designated primary users at a particular time in a particular geographic area.
- Traditional sequences which assume the availability of the entire spectral band with no spectrum hole constraint cannot be applied directly in CR system.

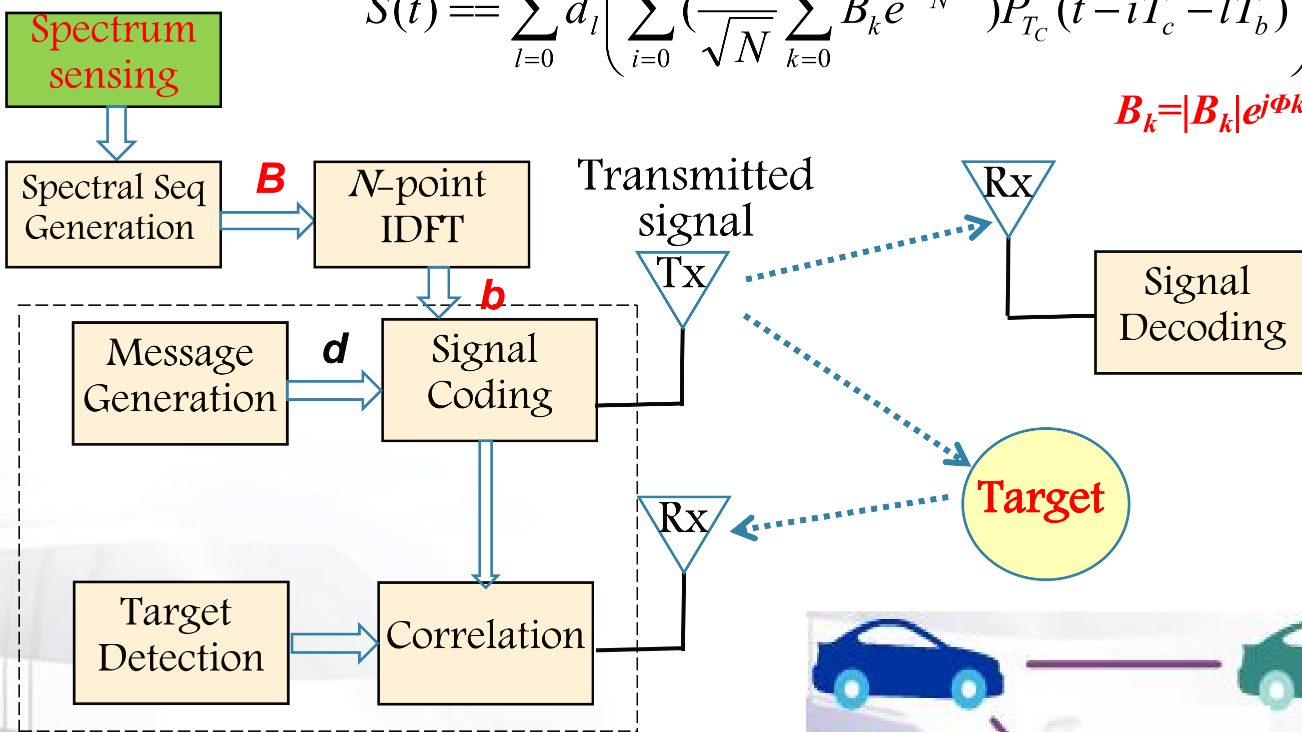
[1] I. F. Akyildiz, W. Y. Lee, et al, "Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey," ***Comput. Netw.***, vol. 50, pp. 2127-2159, Sept. 2006.

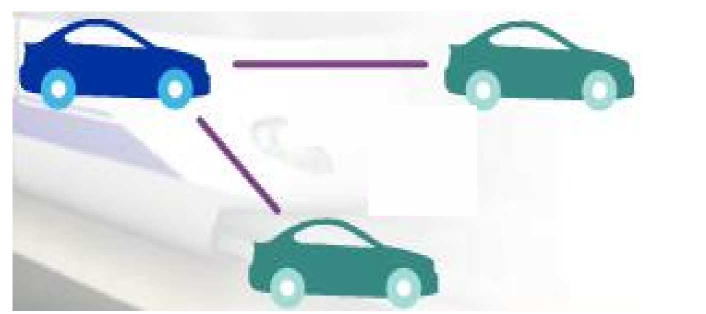
[2] N. Levanon and E. Mozeson, Radar Signals, New York: ***Wiley***, 2004.

System Model of Cognitive Radio

$$S(t) = \sum_{l=0}^{\infty} d_l \left(\sum_{i=0}^{N-1} \left(\frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} B_k e^{j\frac{2\pi}{N}ik} \right) P_{T_C}(t - iT_c - lT_b) \right)$$

$B_k = |B_k| e^{j\Phi k}$, $T_b = NT_c$





Practical Requirements of Seqs Design

- For high power transmission efficiency, it is desirable to have sequences with low **peak-to-average power ratio (PAPR)**.
- In practice, Doppler shifts/spreads, caused by objects such as signal reflectors, moving tx/rx, need to be considered, i.e. **thumbtack shape Ambiguity function (AF)** is desirable.
- For practical applications, sequences satisfying the following conditions are useful:

✓ **Good local AF property in the area of interest;**

✓ **Low PAPR value;**

✓ **Subject to a spectrum hole constraint in frequency domain, ideally, with zero spectral leakage over the spectrum holes.**

[1] S. Hu, Z. L. Liu, et al, "Sequence design for cognitive CDMA communications under arbitrary spectrum hole constraint," *IEEE Journal on Selected Areas in Communications*, vol. 32, pp. 1974-1986, Nov. 2014.
[2] LS. Tsai, et al, "Syntehsizing low autocorrelation and low PAPR OFDM sequences under spectral constraints through convex optimization and GS algorithm," *IEEE Trans. Signal Process.*, vol. 59, pp. 2234-2243, May 2011.

Spectrum Hole, PAPR & AF Constraints

- Assume that there are N subcarriers in the entire spectrum. Let $S=[S_0,S_1,\ldots,S_{N-1}]$ be a subcarrier marking vector, in which $S_k=1$, if the k-th subcarrier is available and 0 otherwise. $\Omega=\{k/S_k=0\}$ is the set of all unavailable subcarrier positions which is also called as a spectrum hole constraint set.
- The PAPR of a time-domain sequence x is defined as

$$\mathrm{PAPR}(x)=10\log_{10}\frac{\max\limits_{0\le i\le N-1}|x_i|^2}{\frac{1}{N}\sum\limits_{i=0}^{N-1}|x_i|^2}$$

For a unimodular sequence, the ideal PAPR equals 0 dB.

- The aperiodic discrete **AF** of frequency-domain sequence X can be defined as

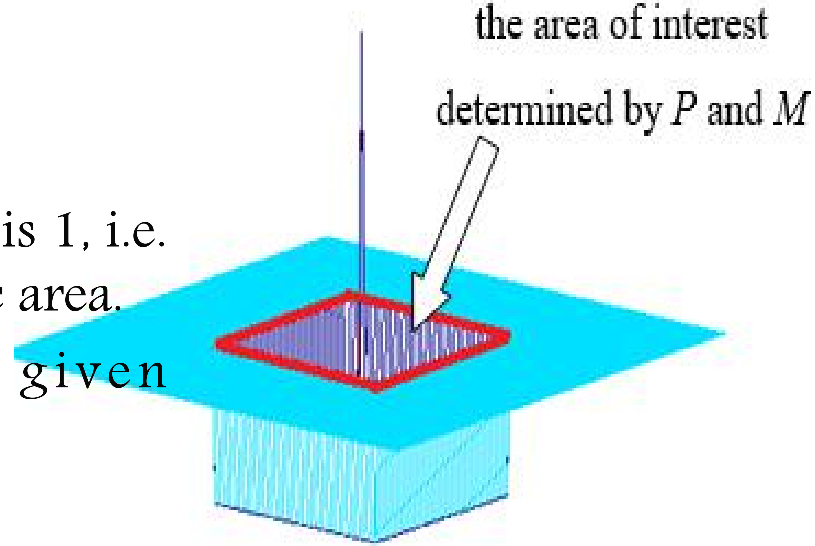
$$\chi_{p,m}=\frac{1}{N}\sum_{i\in I}\left(\left(\sum_{k=0}^{N-1}X_kW_N^{-ik}\right)\left(\sum_{k=0}^{N-1}X_k^*W_N^{(i+m)k}\right)W_N^{-ip}\right)$$

where $I=\{i/-m\le i\le N-1-m\}\cap\{i/0\le i\le N-1\}$.

Joint Resolution in a Specific AF Area

- In radar systems, joint resolution denotes the ratio of the squared magnitude of center peak to the ambiguity surface in the entire delay-Doppler domain.
- The **joint resolution in the specific area of AF plane**

$$\mathrm{Res}(x) = \frac{|\chi_{0,0}|^2}{\sum_{p\in P}\sum_{m\in M}|\chi_{p,m}|^2}$$



The ideal value of joint resolution is 1, i.e. the sidelobes equal 0 in the specific area.

- The **energy term** under a given normalized Doppler shift p

$$E_p = \sum_{m=-N+1}^{N-1}|\chi_{p,m}|^{2q} w_{p,m}$$

$w_{p,m} \in \{0,1\}$ control the shape of the AF sidelobe

Seqs for Optimized AF & PAPR in CR

Let $B=[B_0,B_1,\ldots,B_{N-1}]^T$ be the the frequency-domain sequence with $B_k=|B_k|e^{j\Phi k}$, $B_k=0$, if $k\in\Omega$.

$$\min_{B,X,d} J(B,X,c)=\mu J_1(B,X)+(1-\mu)J_2(B,c)$$

$$\min_{B,X} J_1(B,X)=\left\|B-X\right\|_2^2$$
$$s.t.\ B_k, X_k=0, if\ k\in\Omega$$

$$\min_{B,d} J_2(B,c)=\left\|F_N^H B-c\right\|_2^2$$
$$s.t.\ \text{(i)}\ B_k=0\ if\ k\in\Omega;$$
$$\text{(ii)}\ |c_k|=1,\ k=0,1,\cdots,N-1$$

- J_1 optimizes local AF of the B to be designed.
- J_2 optimizes PAPR of the B to be designed,
- J optimizes PAPR and local AF for a given penalty factor μ, constrained by spectrum hole Ω. (Algorithm 2)
- X is a frequency-domain sequence having good local AF which can be calculated by Energy Gradient Method (Algorithm 1).

Tianjun LIU, Pingzhi FAN, et al, Sequence Design for Optimized Ambiguity Function and PAPR under Arbitrary Spectrum Hole Constraint, , *the 8th Int. Workshop on Signal Design and its Applications in Communications (IWSDA'17)*, 24-28 Sept, 2017, Sapporo, Japan.

Seq Design Example No.1

The number of subcarriers / The length of the sequence	N	64
Entire bandwidth	B	8MHz
Spectrum hole constraint set	Ω	$\{15,16,17,18\} \cup \{45,46,47,48\}$
Unavailable bands		1.875~2.375MHz and 5.625~6.125MHz
Chip duration	T_b	0.125μs
Penalty factor	μ	0.5
Normalized Doppler-shifts of interest set	P	$\{-3,-2,-1,0,1,2,3\}$
Sidelobe control coefficients	$w_{p,m}$	if $\lvert m\rvert <= 10$ and $p \in P$, $w_{p,m}=1$ otherwise 0

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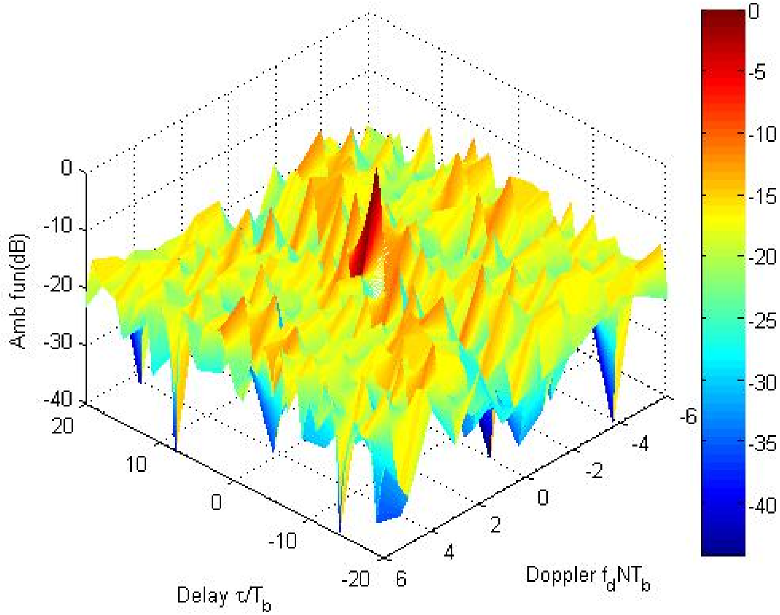
Seq Design Example No.1: AF

The proposed sequences possess larger joint resolution and lower maximum sidelobe in the area of interest, i.e., better local AF performance.

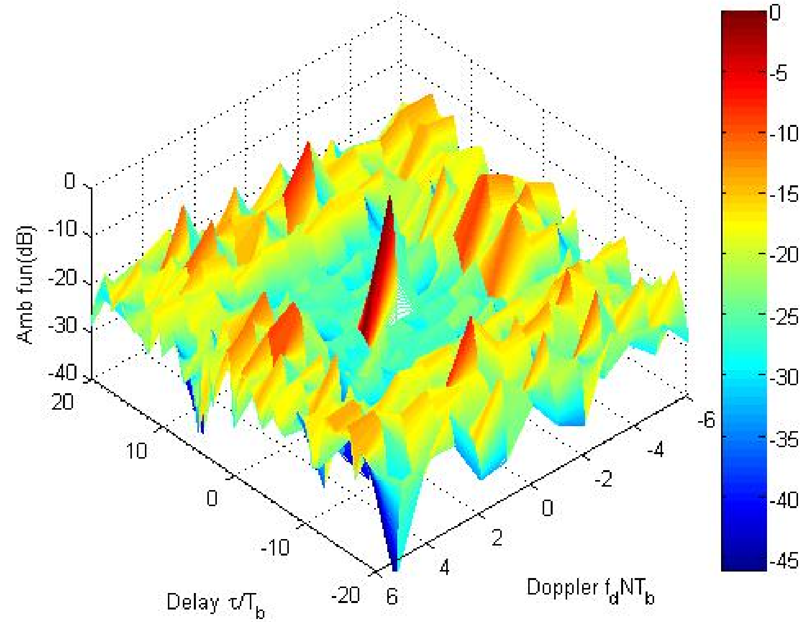


AF of the CR sequence with penalty factor 0.6 [S. Hu, Z. L. Liu, et al, 2014]

PAPR = 2.9403dB

joint resolution=0.32

maximum normalized sidelobe = 0.2993.



AF of the CR sequence generated by the proposed algorithm

PAPR = 2.9603dB

joint resolution = 0.7204

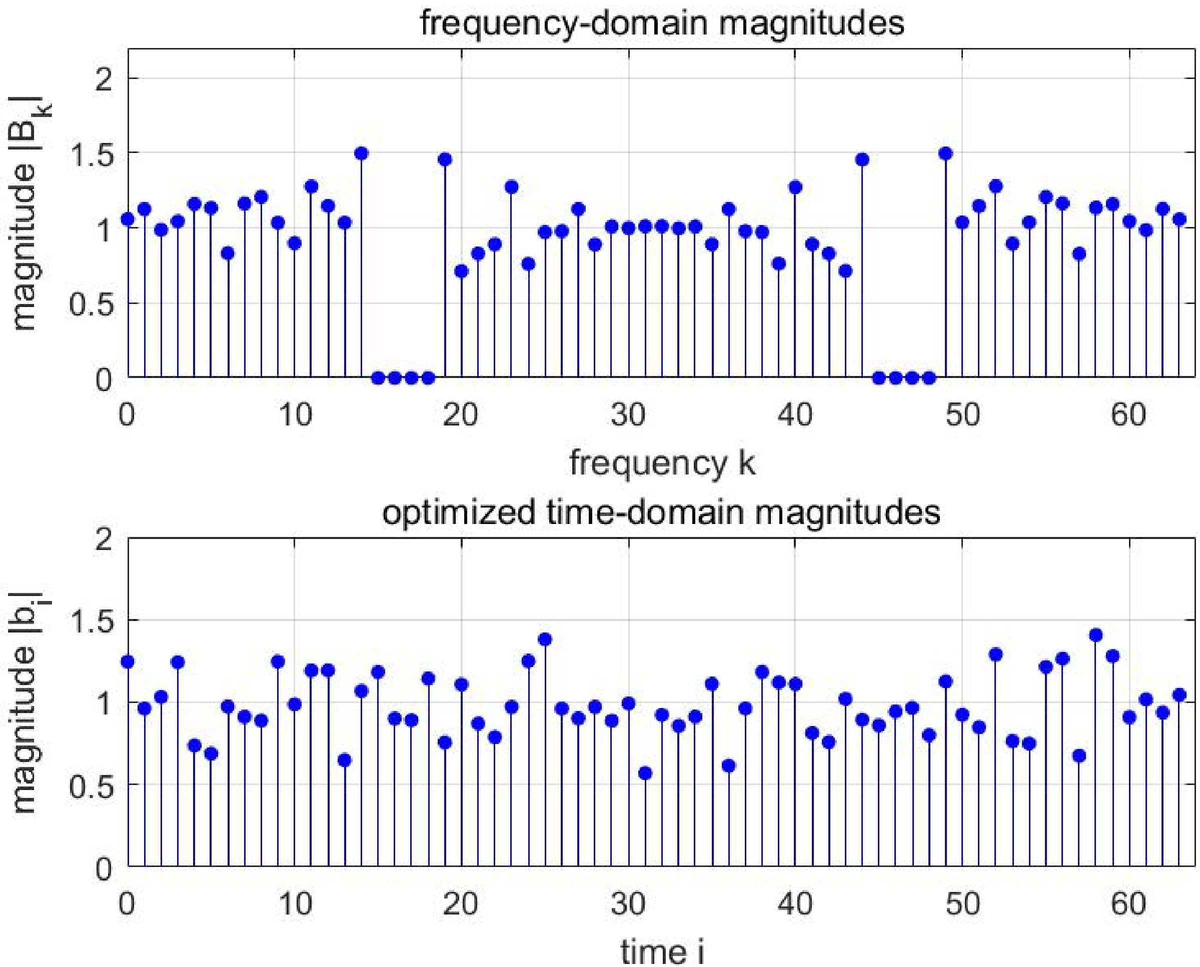
maximum normalized sidelobe = 0.0997.

Seq Design Example No.1: $b_i = |b_i| e^{j\Phi i}$

$$|b| = \begin{bmatrix} 1.2456,0.9621,1.0319,1.2427,0.7367,0.6888, 0.9731,0.9114, \\ 0.8885,1.2457,0.9864,1.1915, 1.1921,0.6488,1.0677,1.1828, \\ 0.9013,0.8910, 1.1431,0.7562,1.1069,0.8700,0.7879,0.9716, \\ 1.2494,1.3807,0.9614,0.9020,0.9719,0.8876, 0.9921,0.5699, \\ 0.9228,0.8563,0.9122,1.1112, 0.6149,0.9622,1.1832,1.1195, \\ 1.1100,0.8129, 0.7579,1.0193,0.8943,0.8591,0.9430,0.9654, \\ 0.8001,1.1248,0.9238,0.8479,1.2897,0.7647, 0.7496,1.2137, \\ 1.2633,0.6748,1.4061,1.2800, 0.9091,1.0158,0.9367,1.0435 \end{bmatrix}$$

$$\phi\{b\} = \begin{bmatrix} 0.0743,0.0412,0.8821,2.5199,0.4484,3.6989,0.4043,3.3764, \\ 2.1680,1.1902,1.6986,6.0612,6.2351,4.6614,3.8043,6.0600, \\ 4.8331,0.0661,1.3119,1.7785,2.1916,4.3442,2.3632,1.4897, \\ 1.7583,0.1366,5.3185,1.9896,5.1492,1.2072,3.3102,0.3972, \\ 2.8009,3.3974,4.2273,4.8842,0.8753,6.2751,4.5530,4.2893, \\ 3.7769,2.0628,5.1391,1.9194,4.2604,5.5966,2.3147,5.1521, \\ 4.5880,1.1147,1.0458,1.6849,2.2452,1.2186,0.4693,6.2549, \\ 2.4806,5.1376,2.3195,0.3942,6.0375,3.1529,5.0830,5.7712 \end{bmatrix}$$

Seq Design Example No.1: $|B_k|$ & $|b_i|$



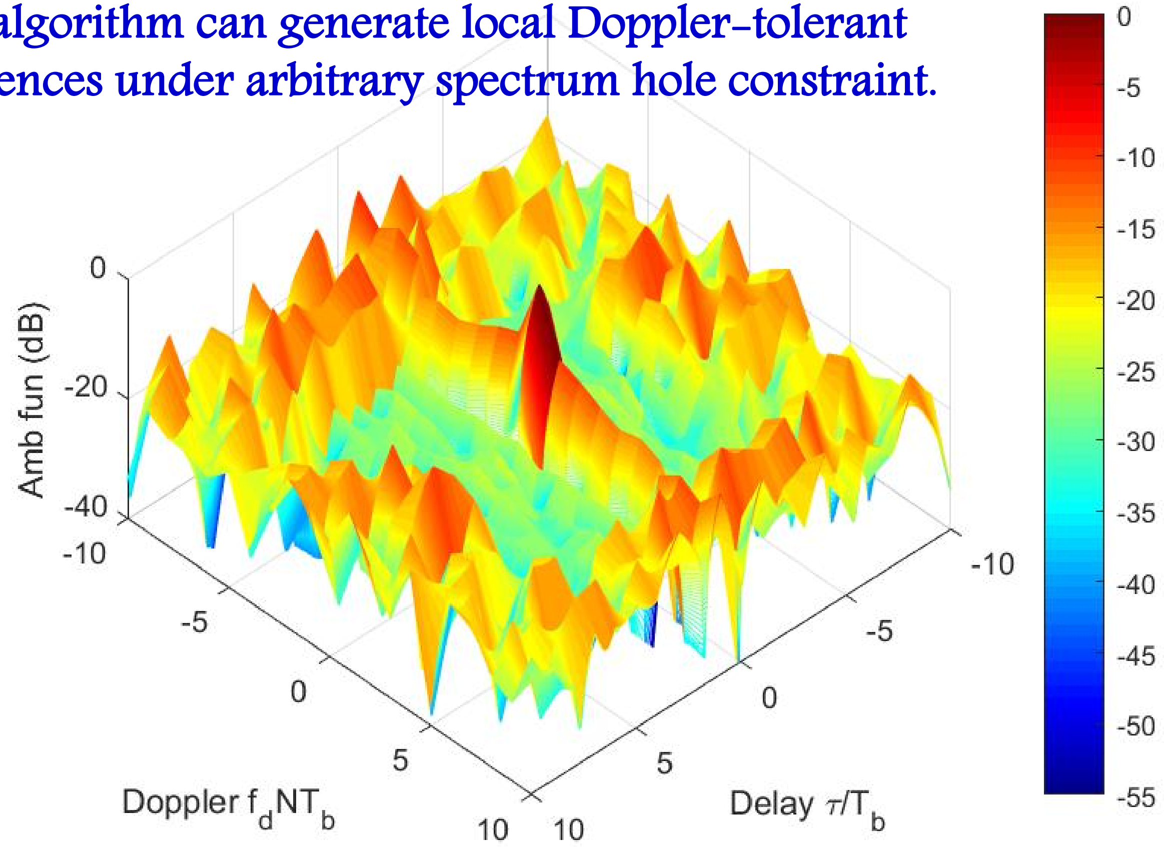
Frequency- and time-domain magnitudes of the CR sequence

Seq Design Example No.2

The number of subcarriers / The length of the sequence	N	64
Entire bandwidth	B	8MHz
The spectrum hole constraint set	Ω	{0} ∪ {27,28,…,37}
Unavailable bands		0~0.125MHz and 3.375~4.75MHz
Chip duration	T_b	0.125μs
Penalty factor	μ	0.95
Normalized Doppler-shifts of interest set	P	{-6,-5,…,0,…,5,6}
Sidelobe control coefficients	$w_{p,m}$	if |m|<=5, m≠0, and p∈P, $w_{p,m}$=1 otherwise 0

Seq Design Example No.2: AF

The algorithm can generate local Doppler-tolerant sequences under arbitrary spectrum hole constraint.



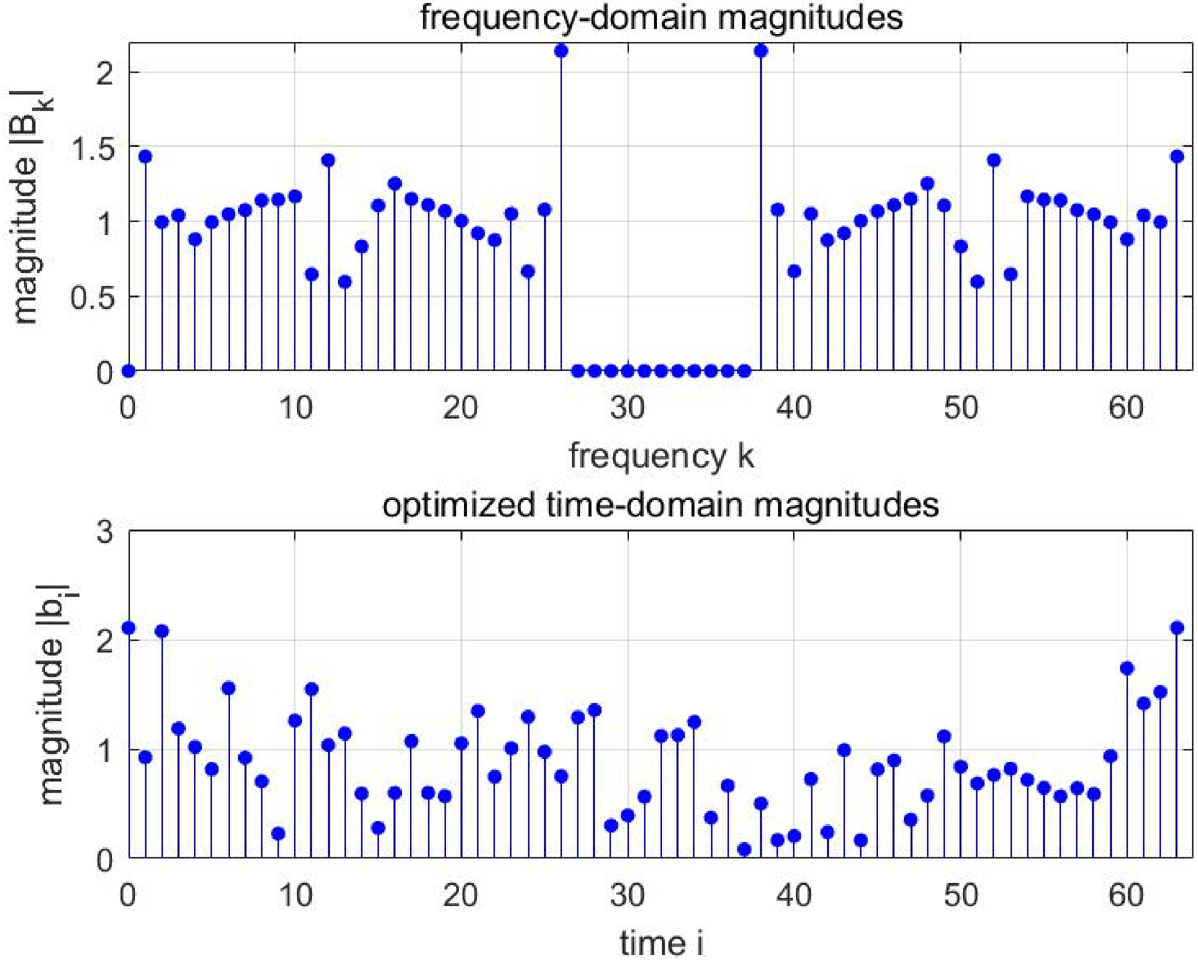
AF of the CR sequence (local Doppler-tolerant sequence) generated by the algorithm 2, PAPR = 6.4684dB

Seq Design Example No.2: $b_i = |b_i| e^{j\Phi i}$

$$
|b| = \begin{bmatrix}
2.1047,0.9234,2.0737,1.1864,1.0182,0.8156,1.5545,0.9185, \\
0.7040,0.2281,1.2580,1.5463,1.0345,1.1407,0.5914,0.2795, \\
0.5985,1.0708,0.5990,0.5701,1.0516,1.3467,0.7468,1.0062, \\
1.2932,0.9742,0.7507,1.2877,1.3542,0.3018,0.3939,0.5658, \\
1.1192,1.1270,1.2450,0.3723,0.6649,0.0837,0.5018,0.1683, \\
0.2053,0.7262,0.2410,0.9892,0.1662,0.8135,0.8956,0.3551, \\
0.5746,1.1153,0.8378,0.6859,0.7619,0.8183,0.7198,0.6458, \\
0.5687,0.6424,0.5878,0.9346,1.7361,1.4141,1.5186,2.1058
\end{bmatrix}
$$

$$
\phi\{b\} = \begin{bmatrix}
2.2730,5.0365,2.9347,1.9773,1.4428,3.7142,5.3943,1.0365, \\
6.1814,0.4459,0.8328,6.1770,3.7039,4.8468,0.0524,1.9966, \\
6.1954,1.0657,0.1097,2.1503,0.4612,5.9217,4.0226,5.5063, \\
1.2856,1.8026,3.3696,2.2454,0.5400,0.8486,5.2058,0.5774, \\
2.3447,4.2957,3.6805,2.4241,0.2465,5.1021,5.0601,3.1556, \\
3.1206,2.8080,3.7402,5.7264,5.3765,4.2162,3.2305,2.4128, \\
3.8151,3.4119,6.0074,0.2876,4.2889,4.2543,3.0302,3.9566, \\
5.3515,5.7836,2.7042,5.3559,4.6889,3.4424,0.0885,1.7605
\end{bmatrix}
$$

Seq Design Example No.2: $|B_k|$ & $|b_i|$



Frequency- and time-domain magnitudes of the CR sequence

Concluding Remarks

- One can construct a Doppler resilient pulse train of Golay waveforms, as well as Z-Golay waveforms.
- Z-Golay codes exist for all lengths, as well as having better Doppler tolerance within the zero correlation zone.
- We can also achieve the same Doppler tolerance based ESP Z-Golay waveforms.
- One can also design Doppler-resilient codes in neighborhood of rational $\theta=2\pi i/m$, by oversampling PTM or ESP codes.
- It is possible to design sequences with desirable AF & PAPR constrained by spectrum hole in cognitive radio.

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Questions?

请斧正，谢谢！